Deep learning preliminaries

STAT 4710

November 14, 2023

Rolling into a new unit!

- **Unit 1:** R for data mining
- **Unit 2:** Prediction fundamentals
- **Unit 3:** Regression-based methods
- **Unit 4:** Tree-based methods
- **Unit 5:** Deep learning

Lecture 1: Deep learning preliminaries

Lecture 2: Neural networks

Lecture 3: Deep learning for images

Lecture 4: Deep learning for text

Lecture 5: Unit review and quiz in class

What is deep learning?

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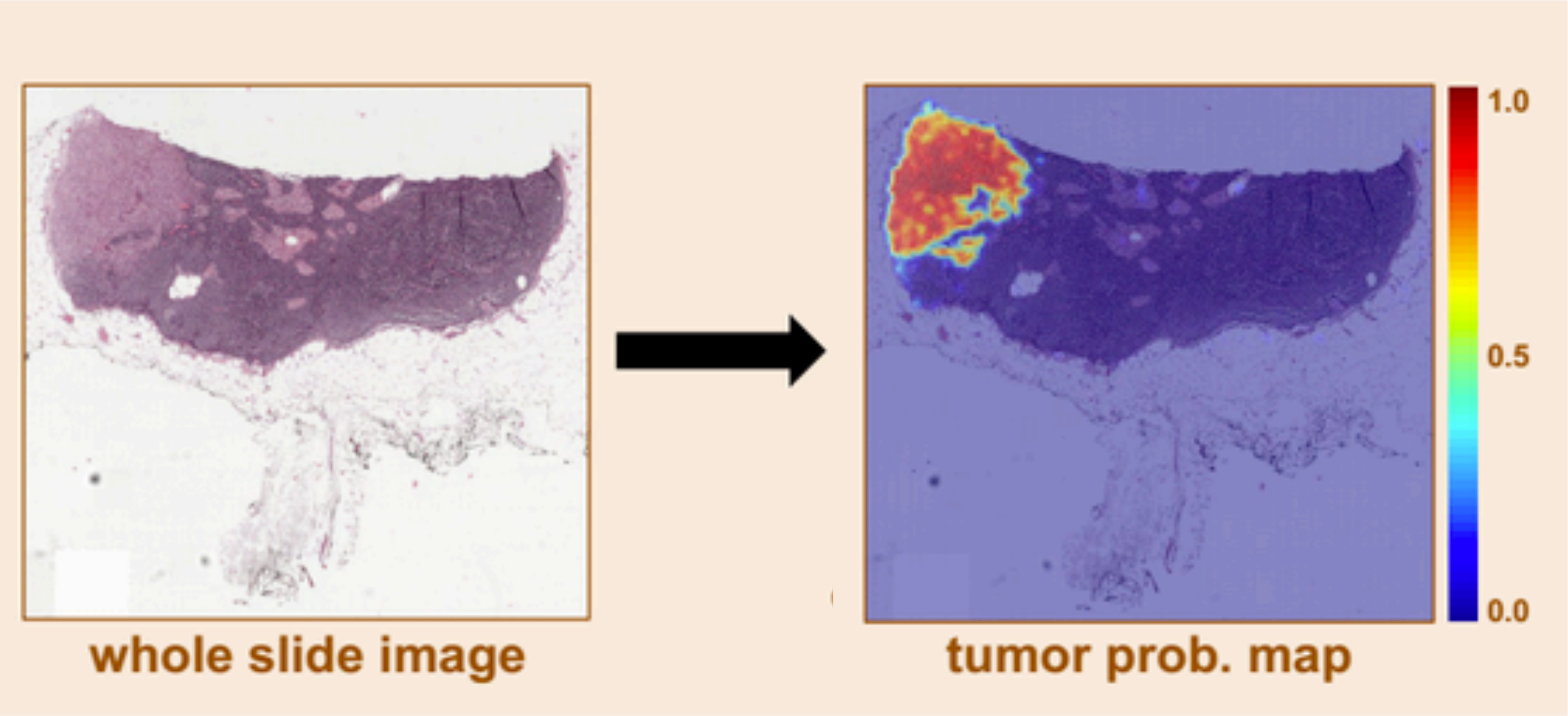

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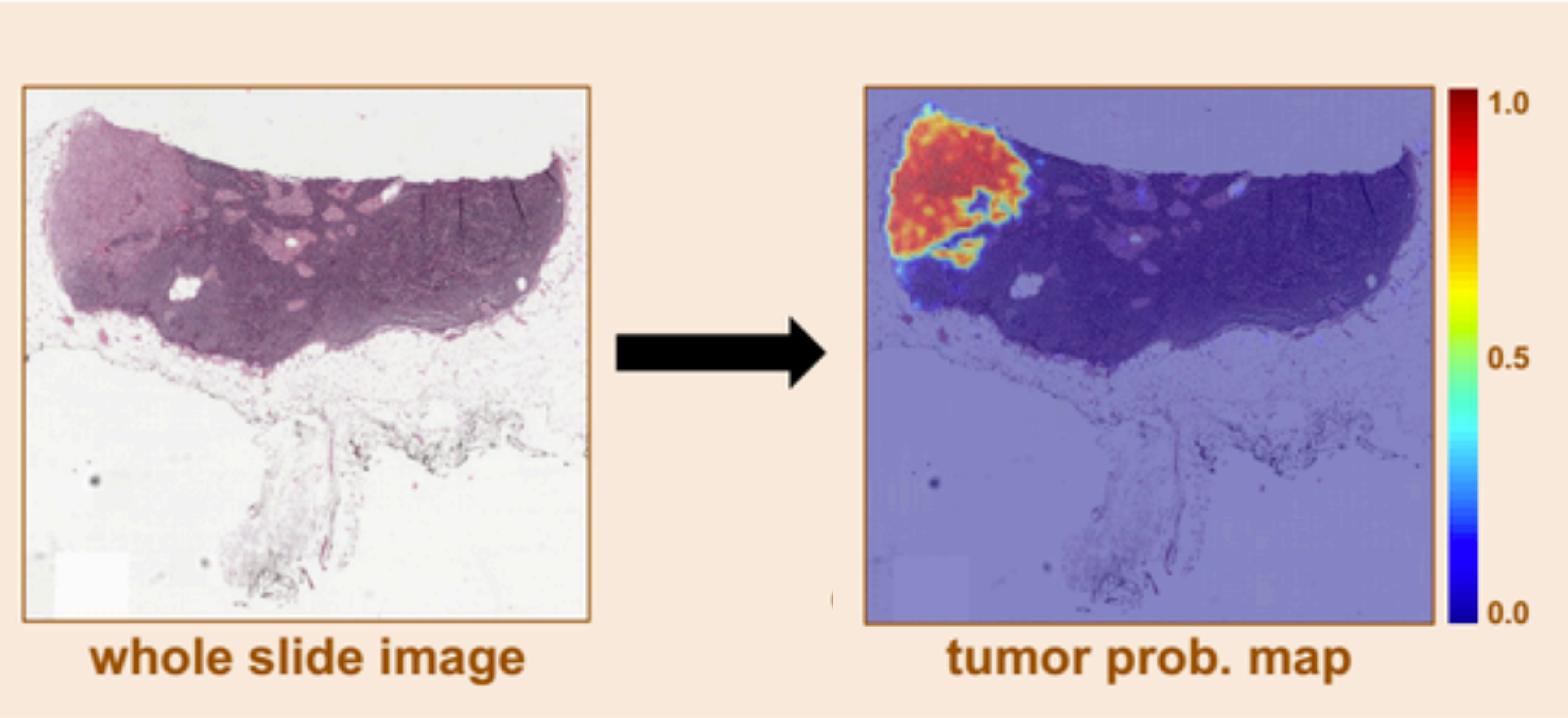

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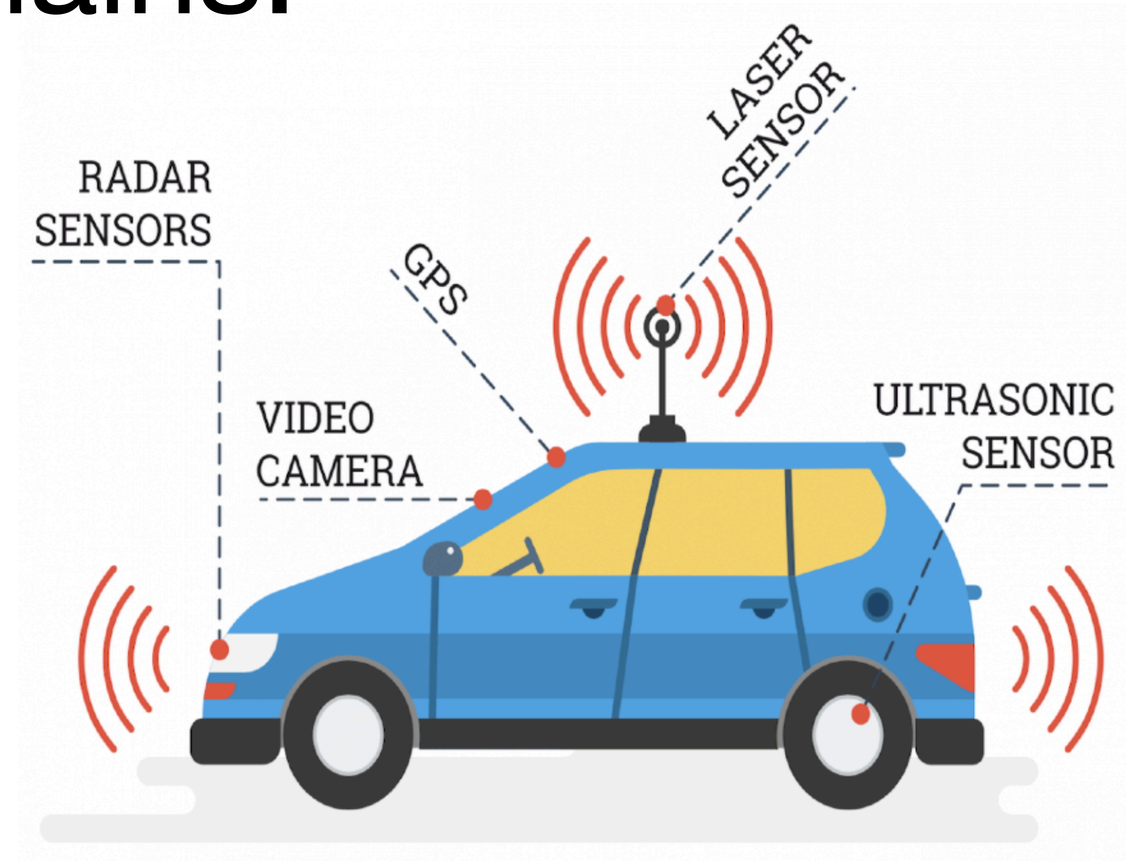

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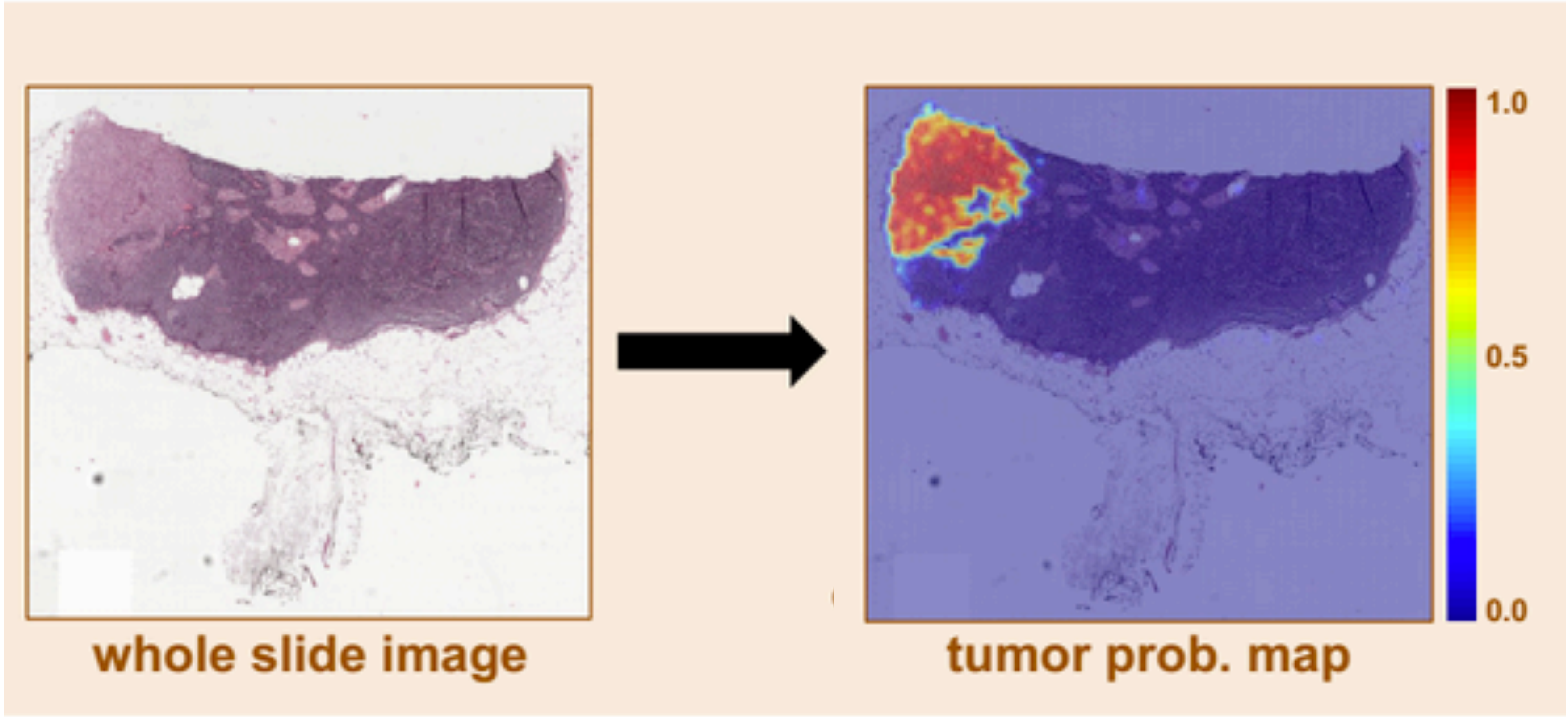

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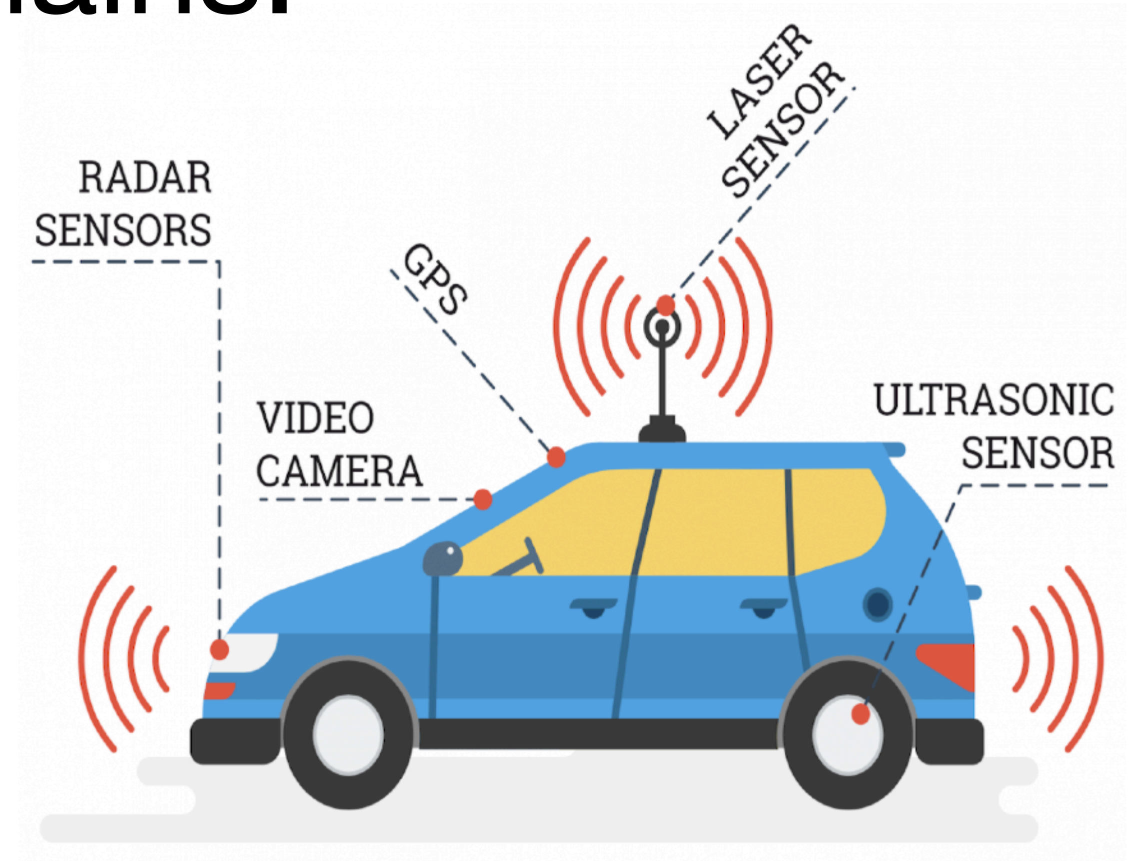

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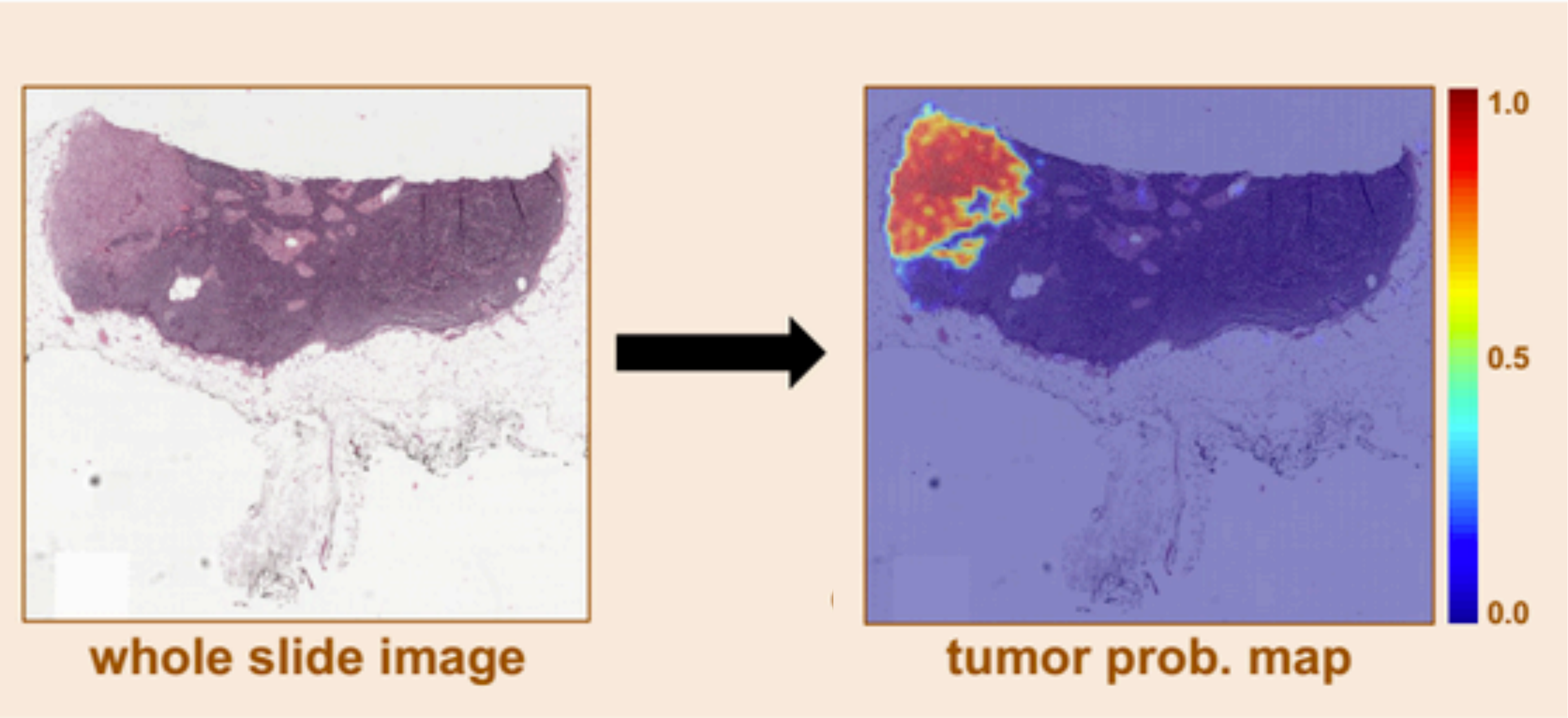

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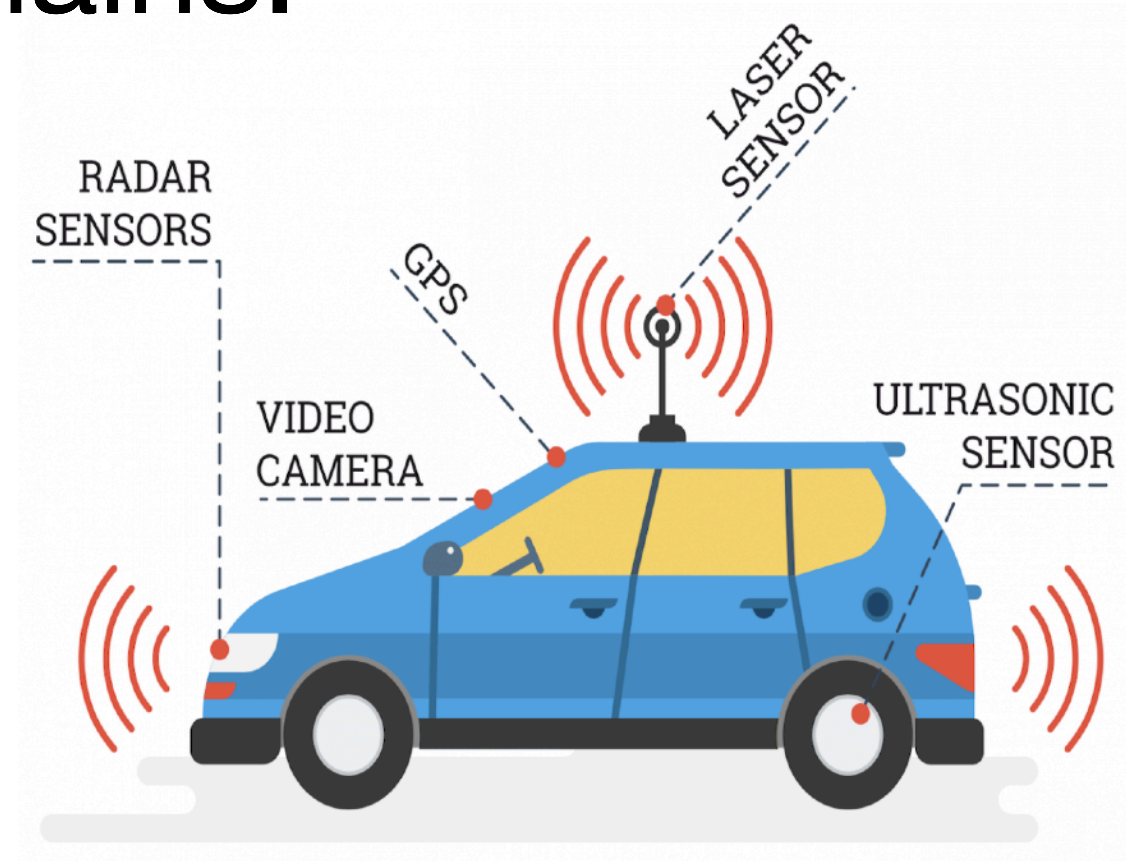

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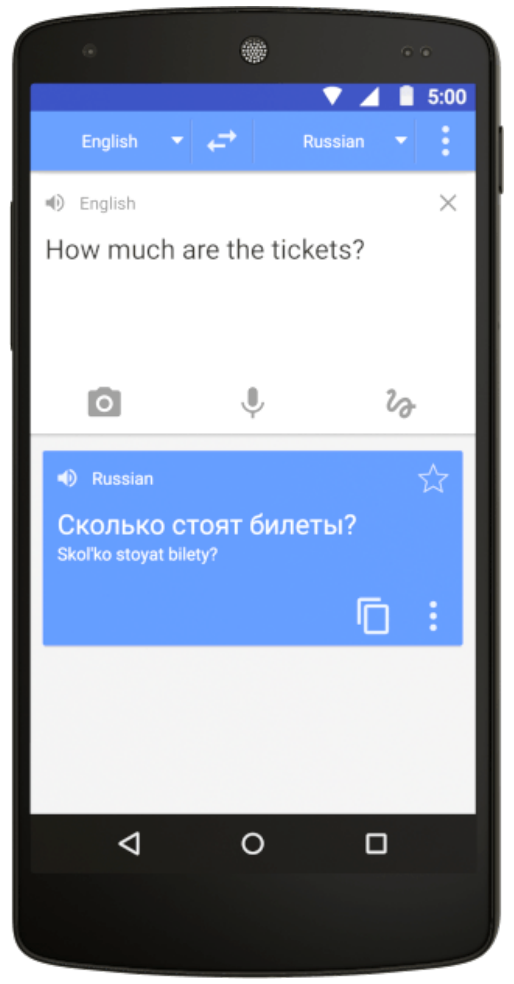

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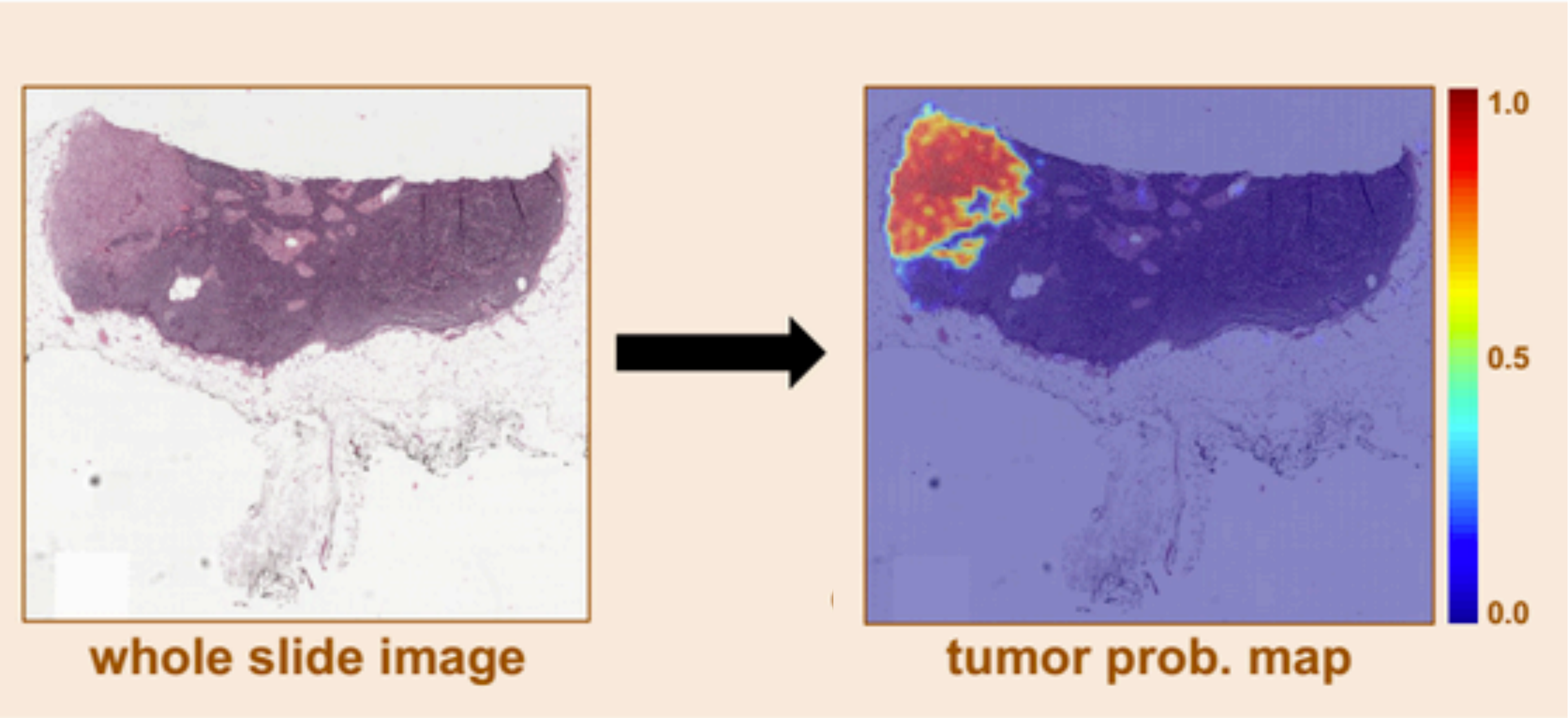

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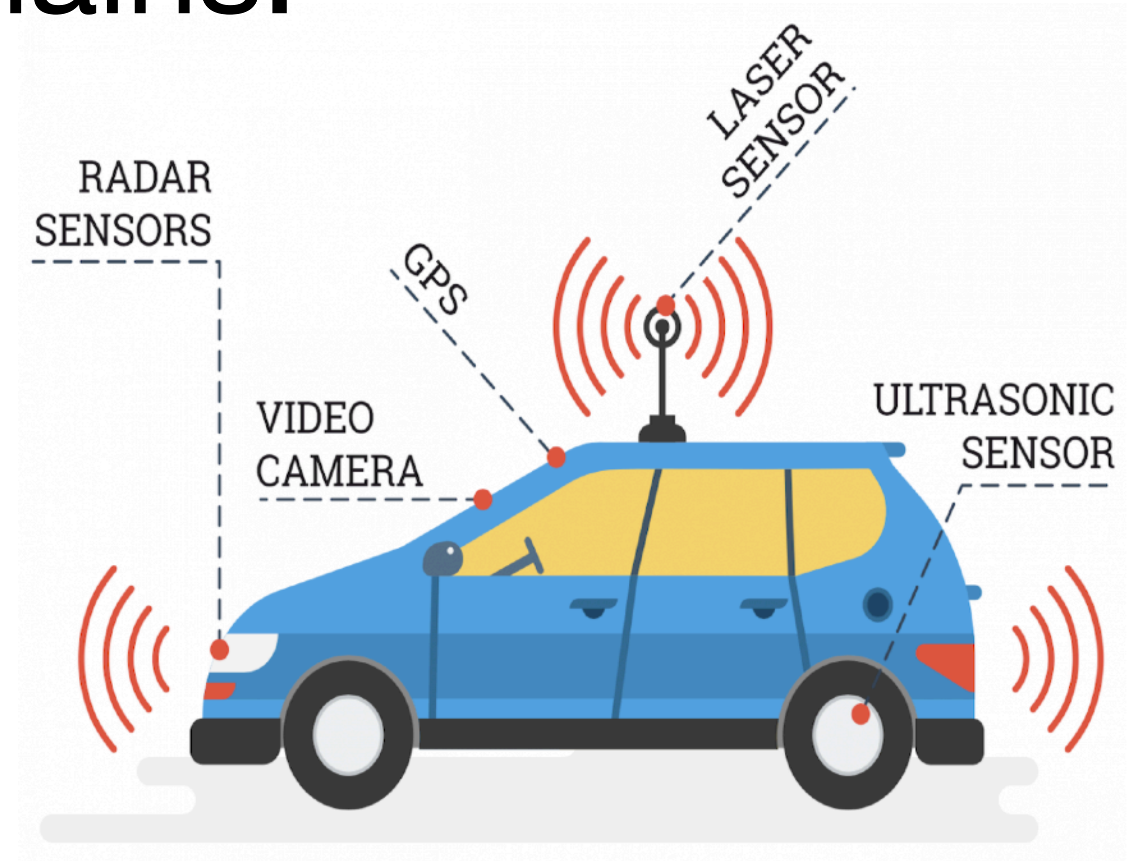

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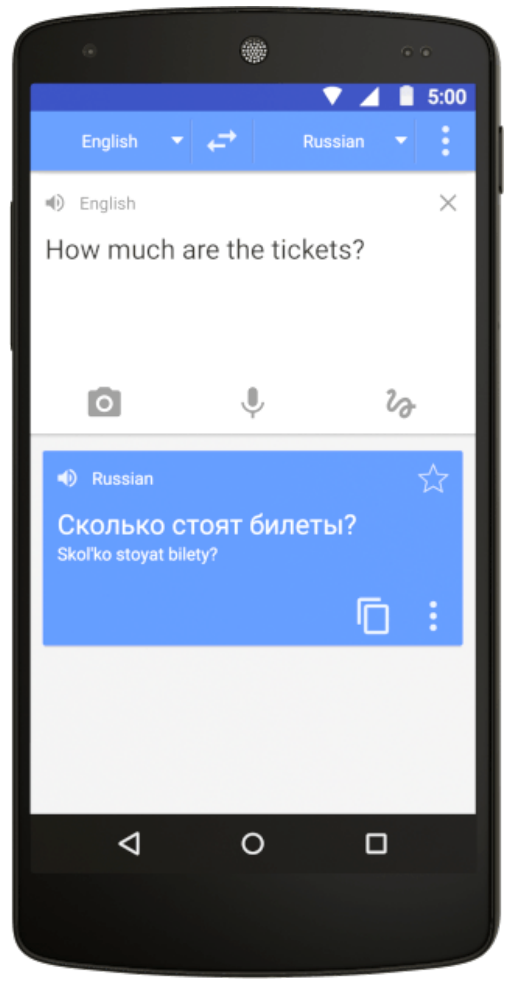

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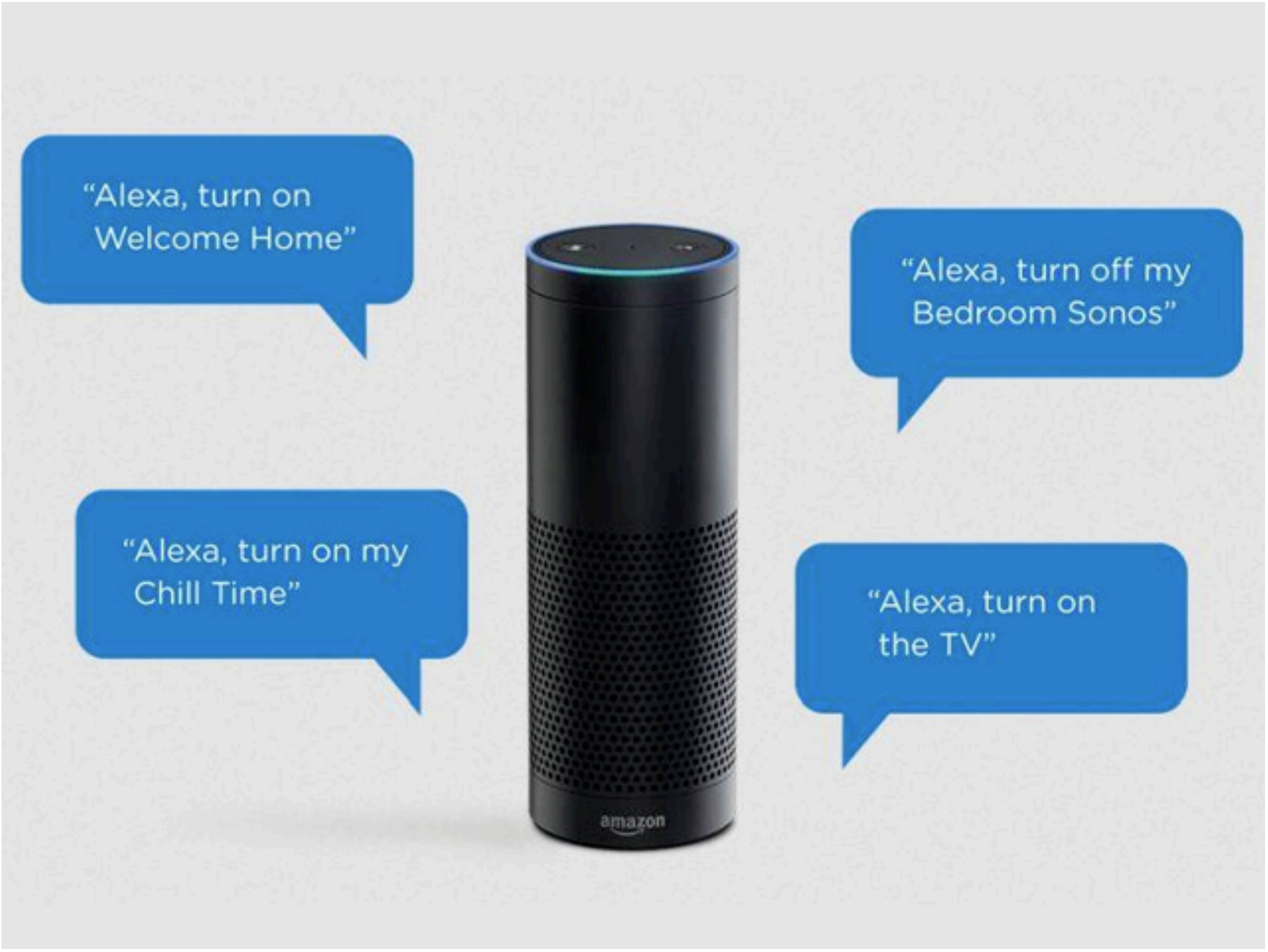

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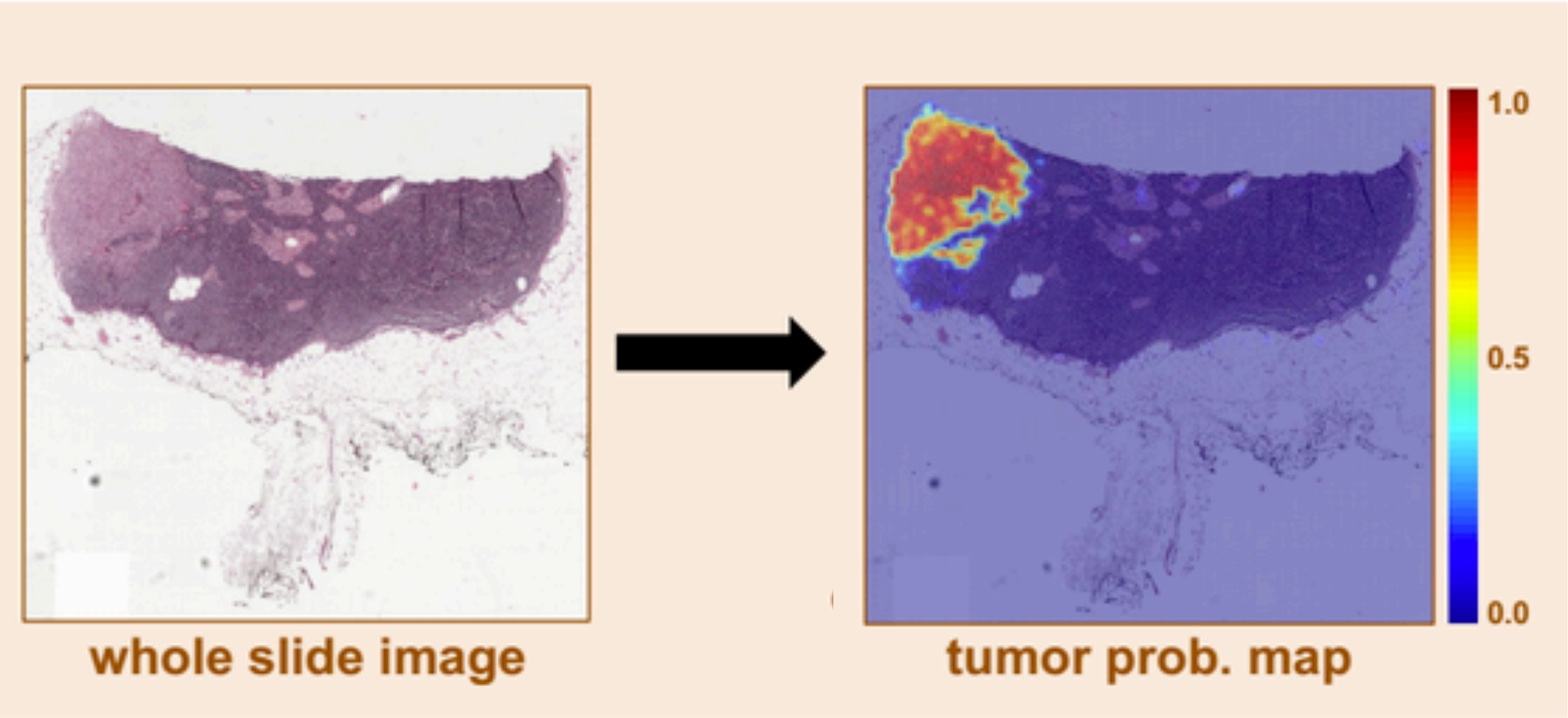

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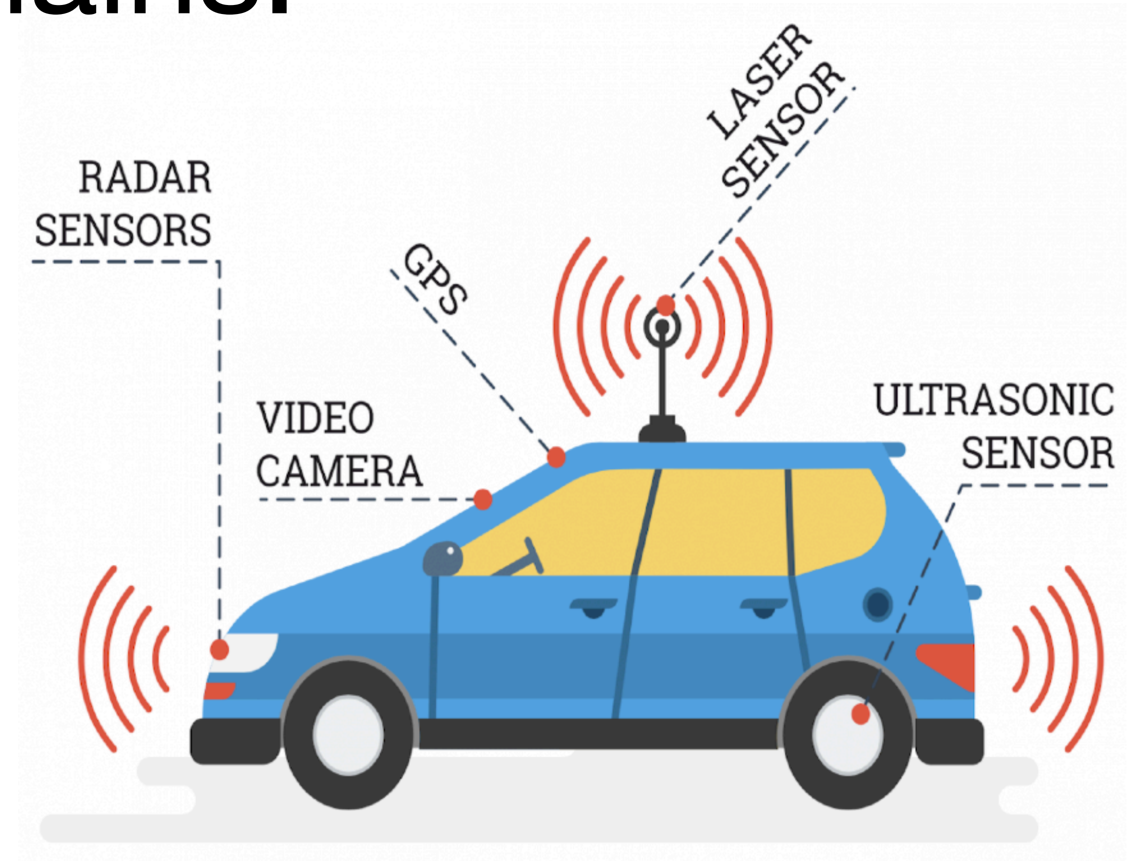

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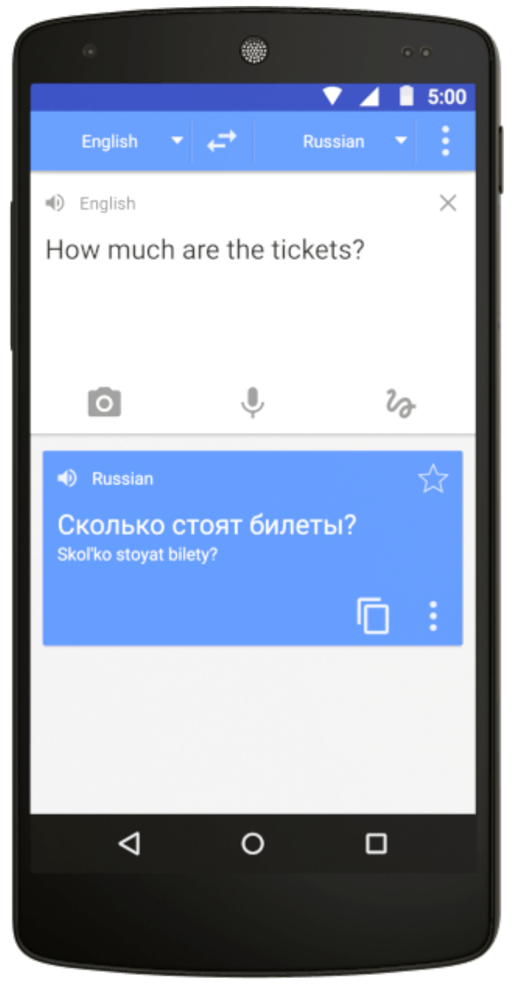

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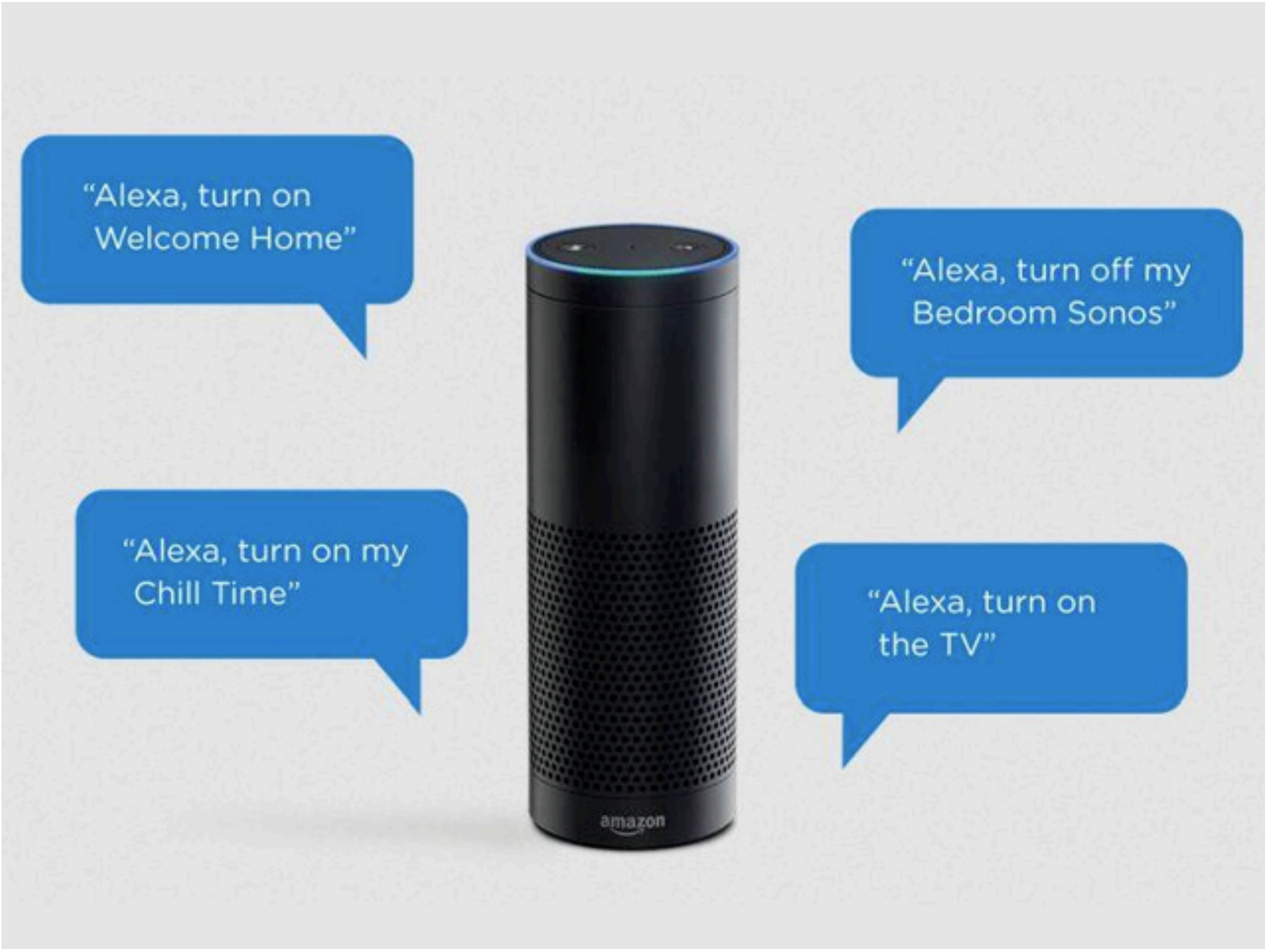

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Game plan for Unit 5

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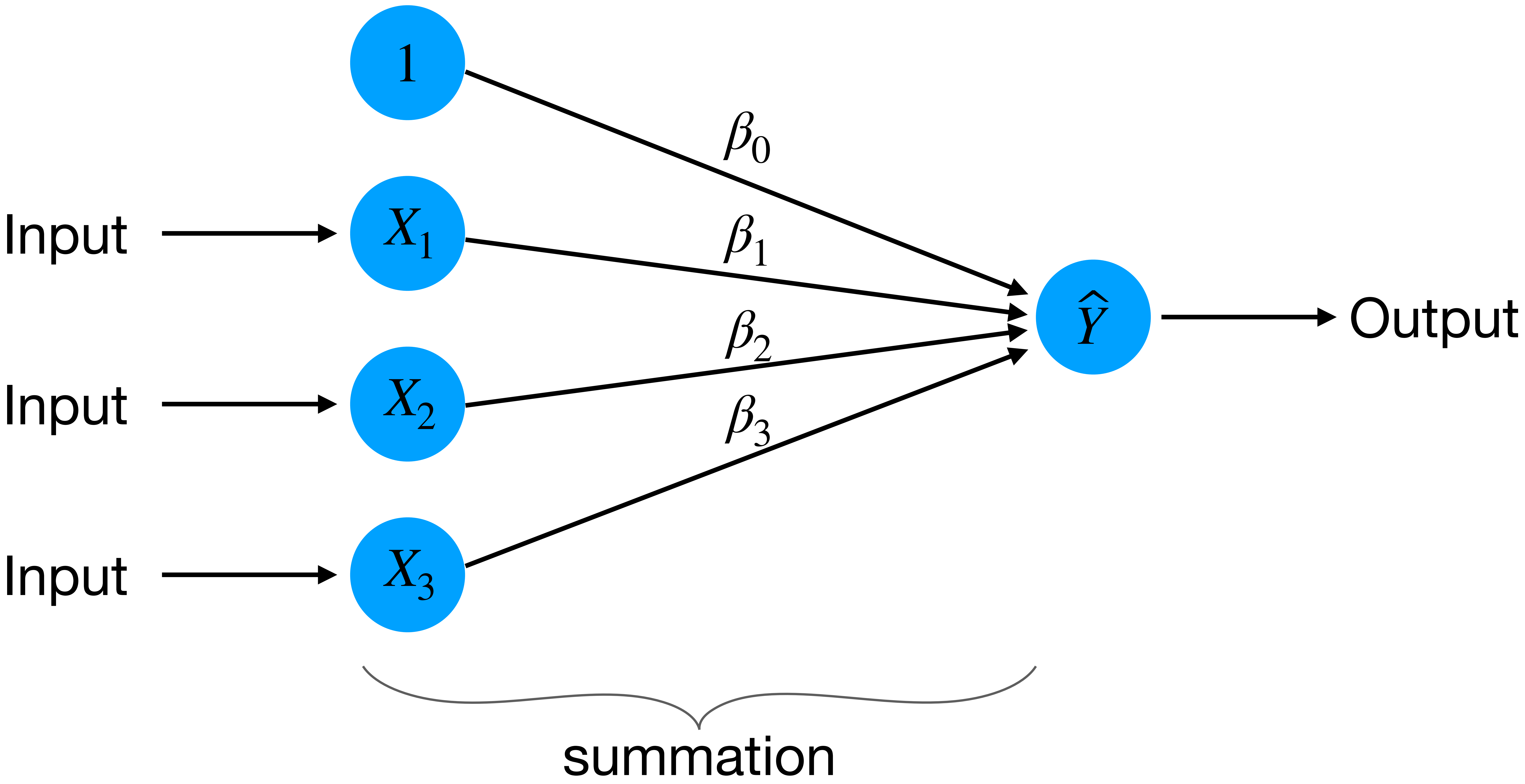
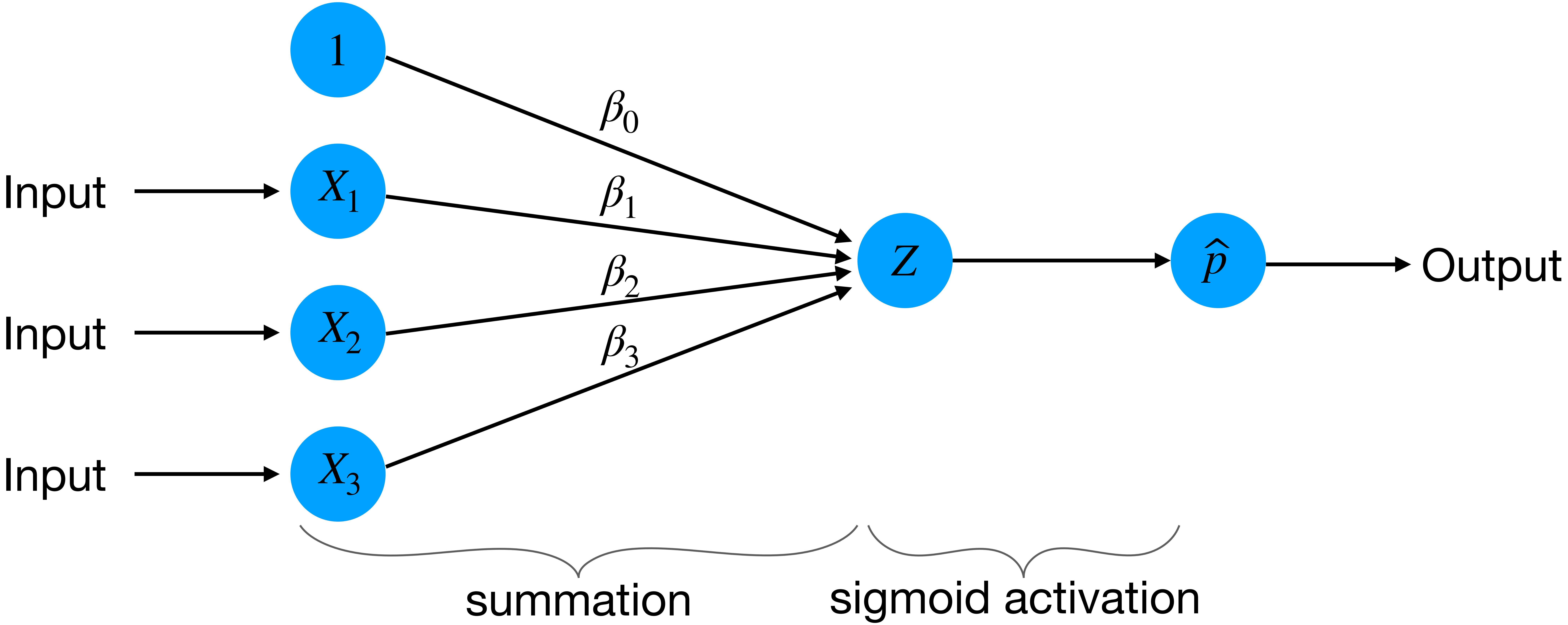
Models as graphs: Linear regression

$$\widehat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$



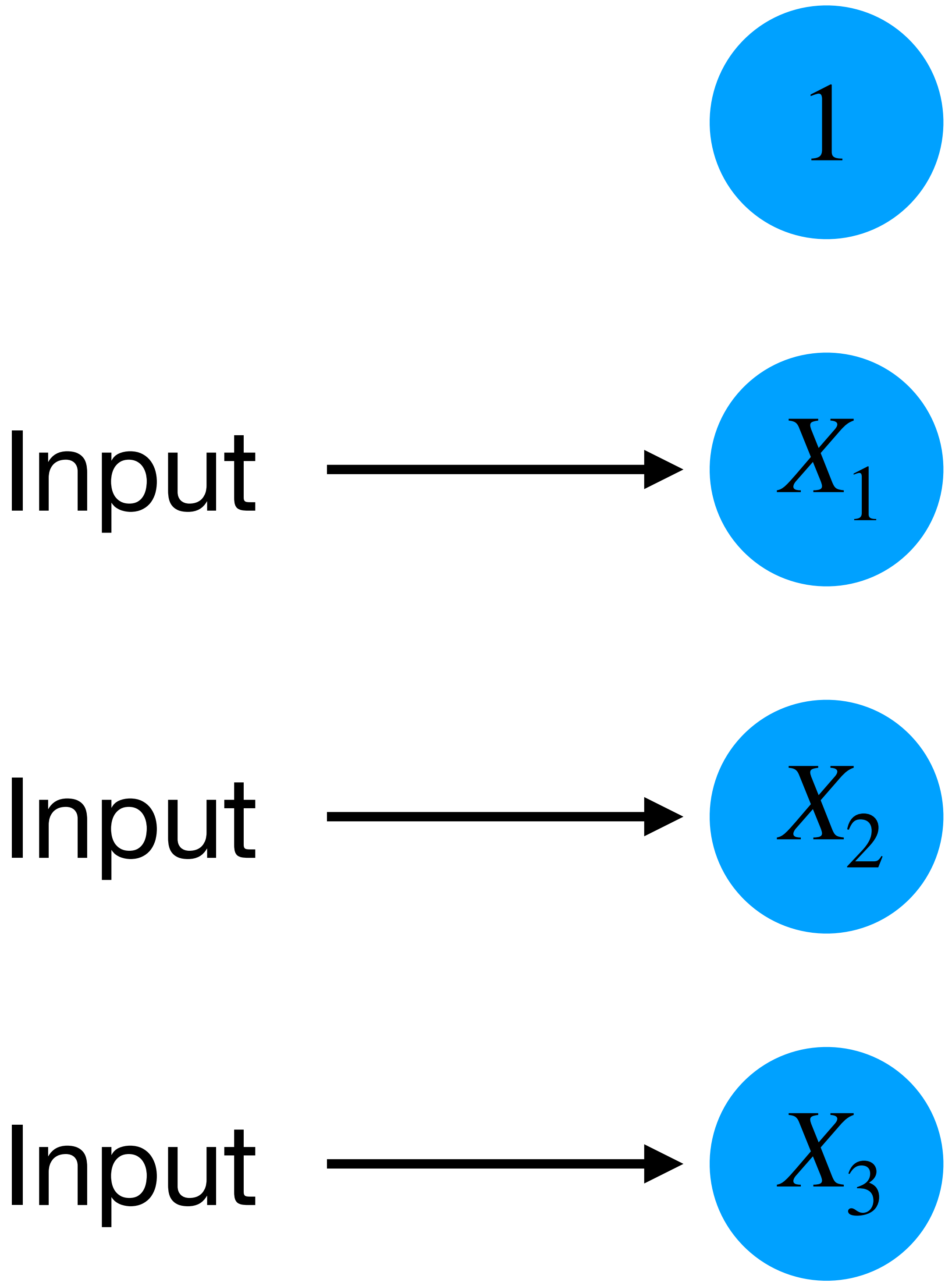
Models as graphs: Logistic model

$$Z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3; \quad \hat{p} = \text{logistic}(Z) = \frac{e^Z}{1 + e^Z}$$



Models as graphs: Multi-class logistic model

Suppose the response has more than two levels.



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Suppose the response has more than two levels.

1

Input $\rightarrow X_1$

Input $\rightarrow X_2$

Input $\rightarrow X_3$

Image is flattened to get vector of input features

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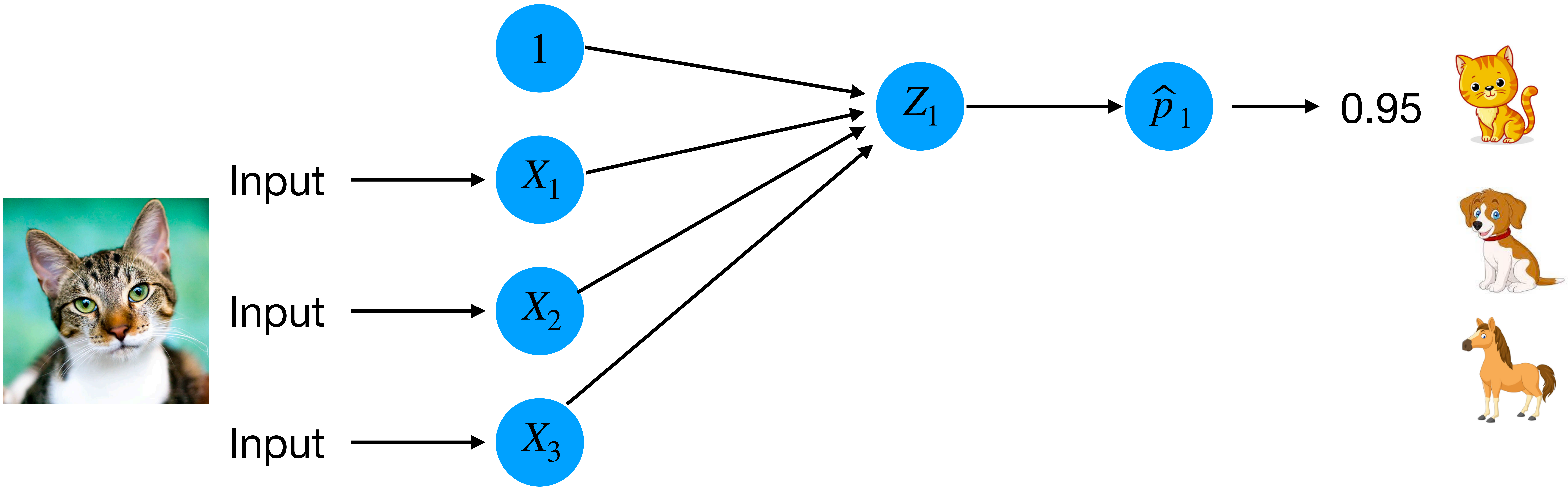

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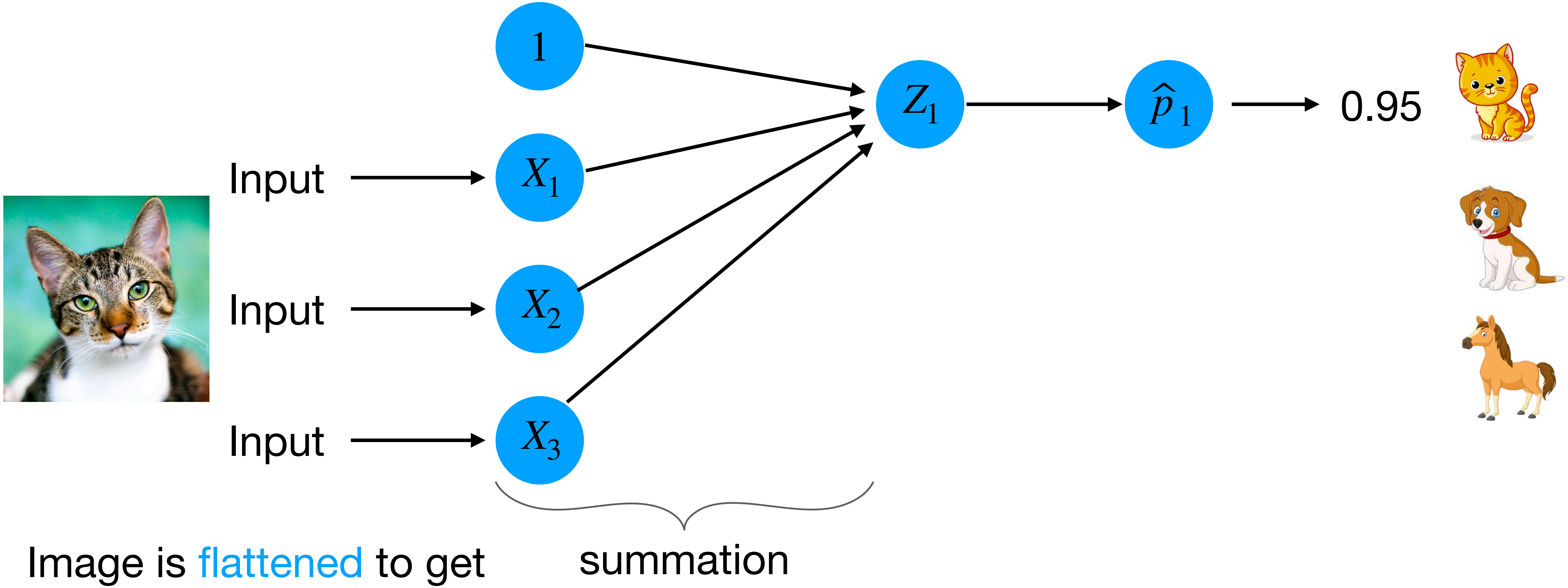

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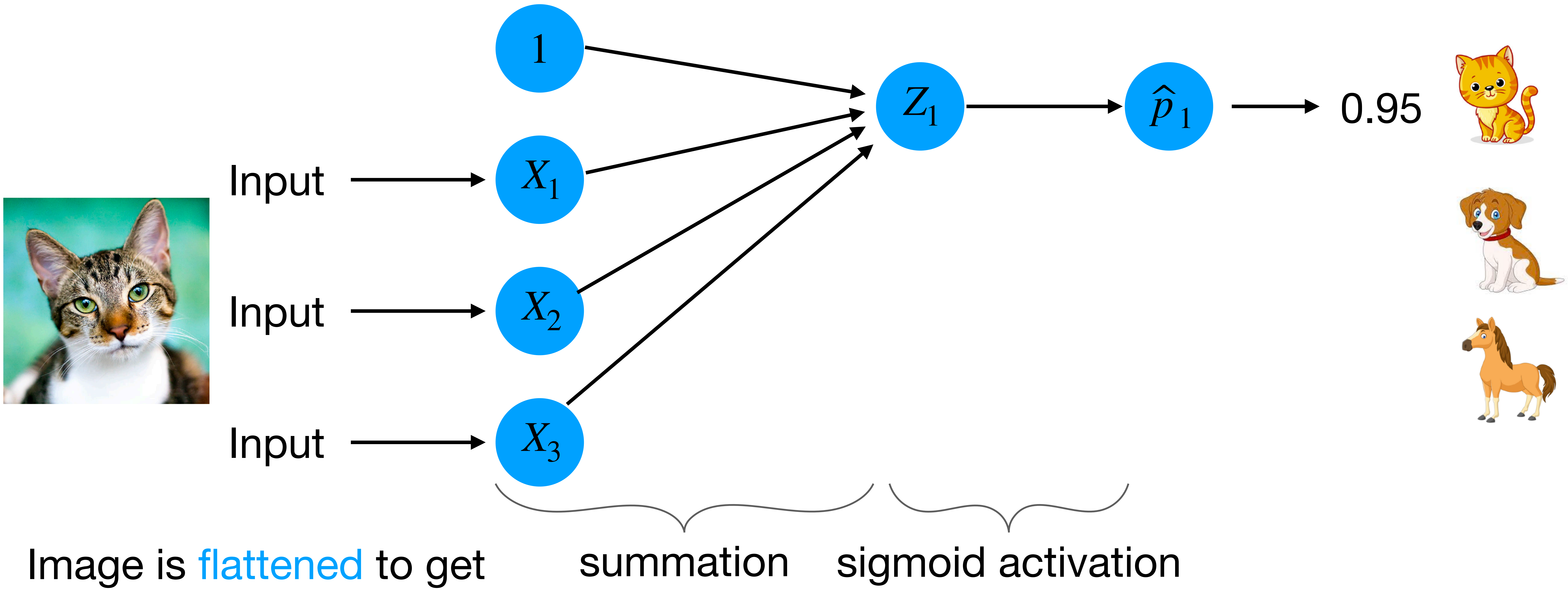

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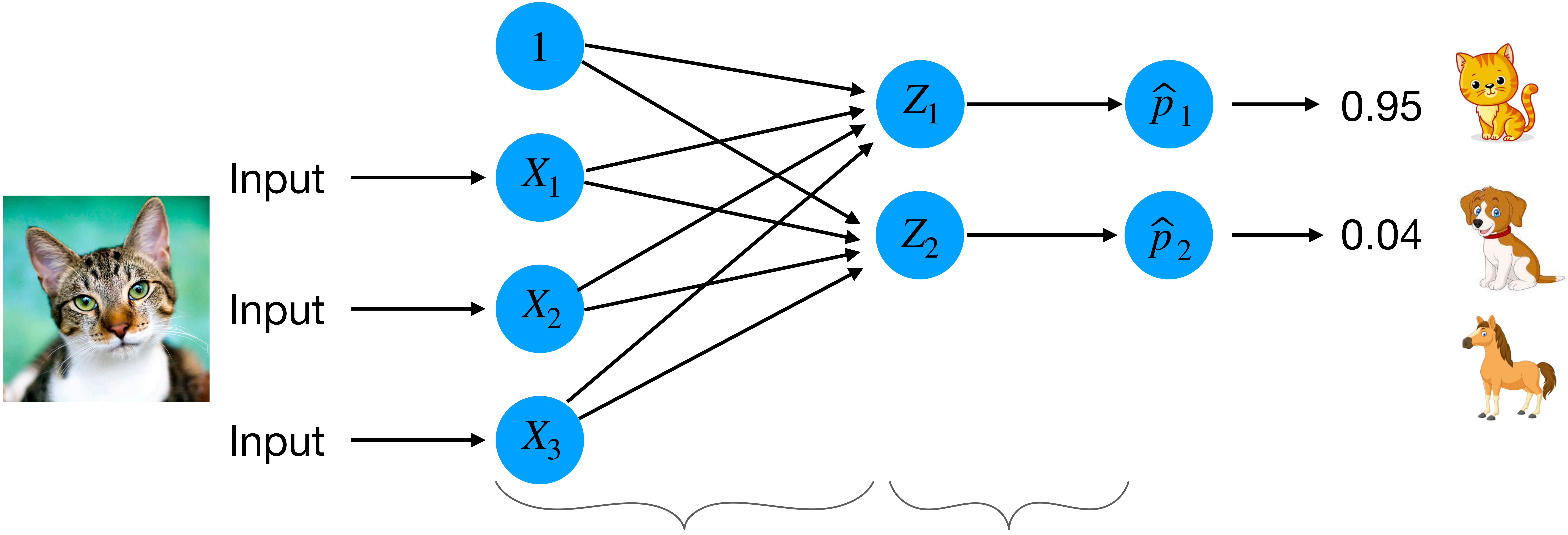

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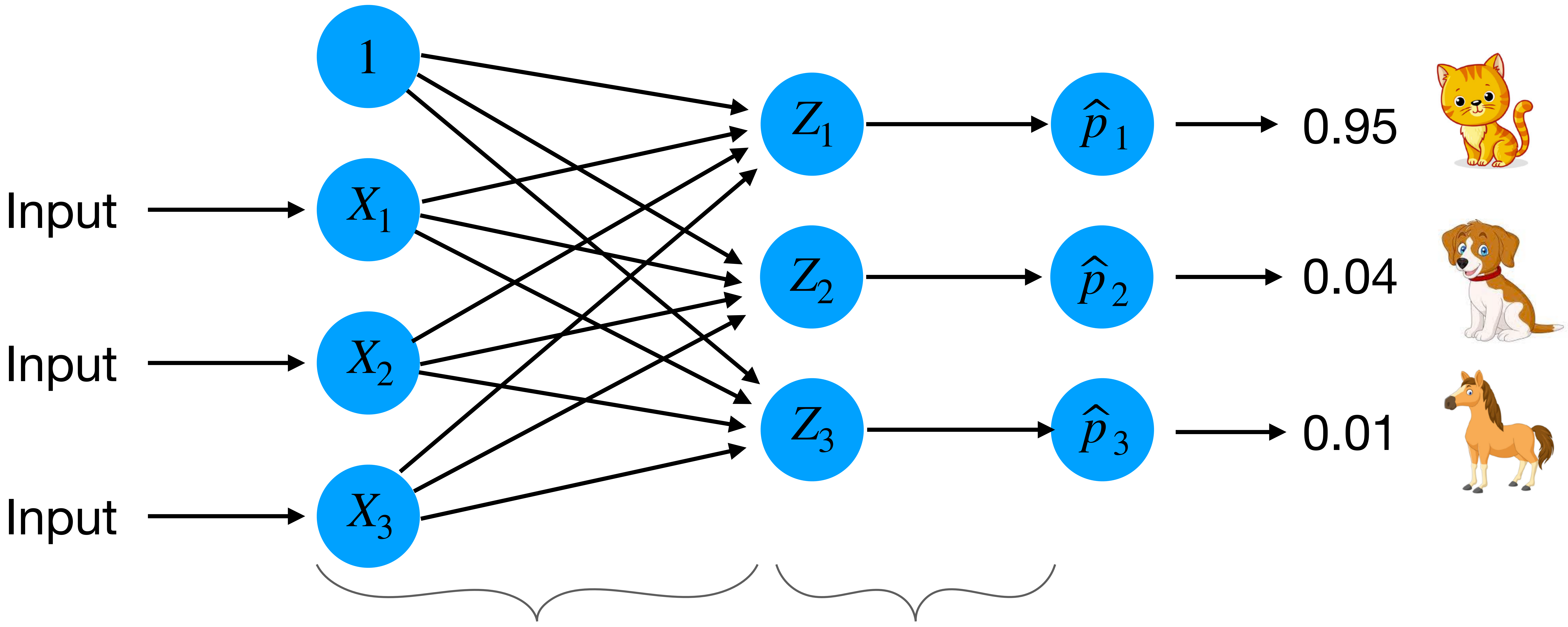

summation sigmoid activation

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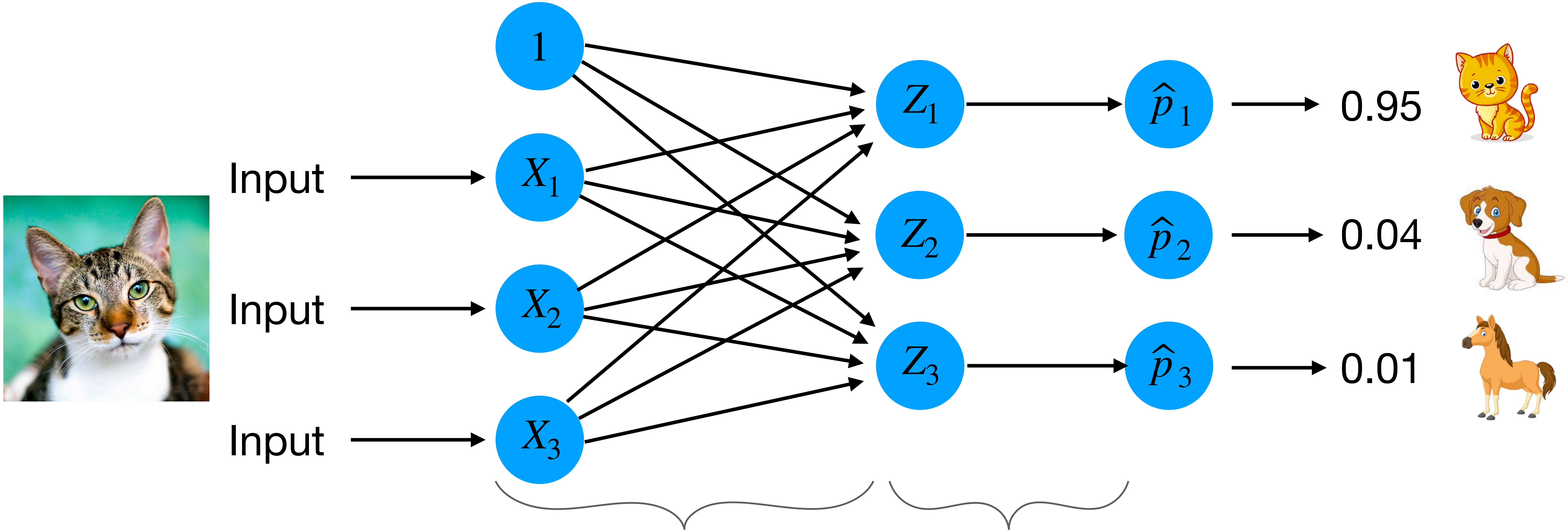

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summation (fully connected layer)

sigmoid activation

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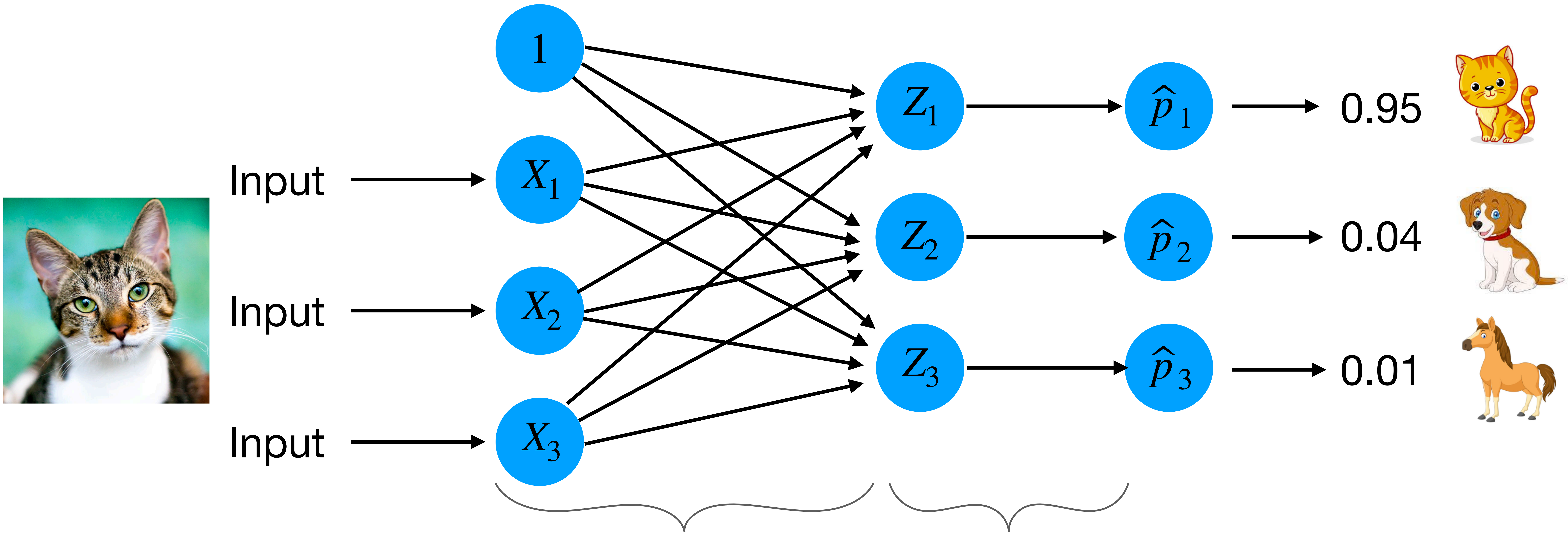

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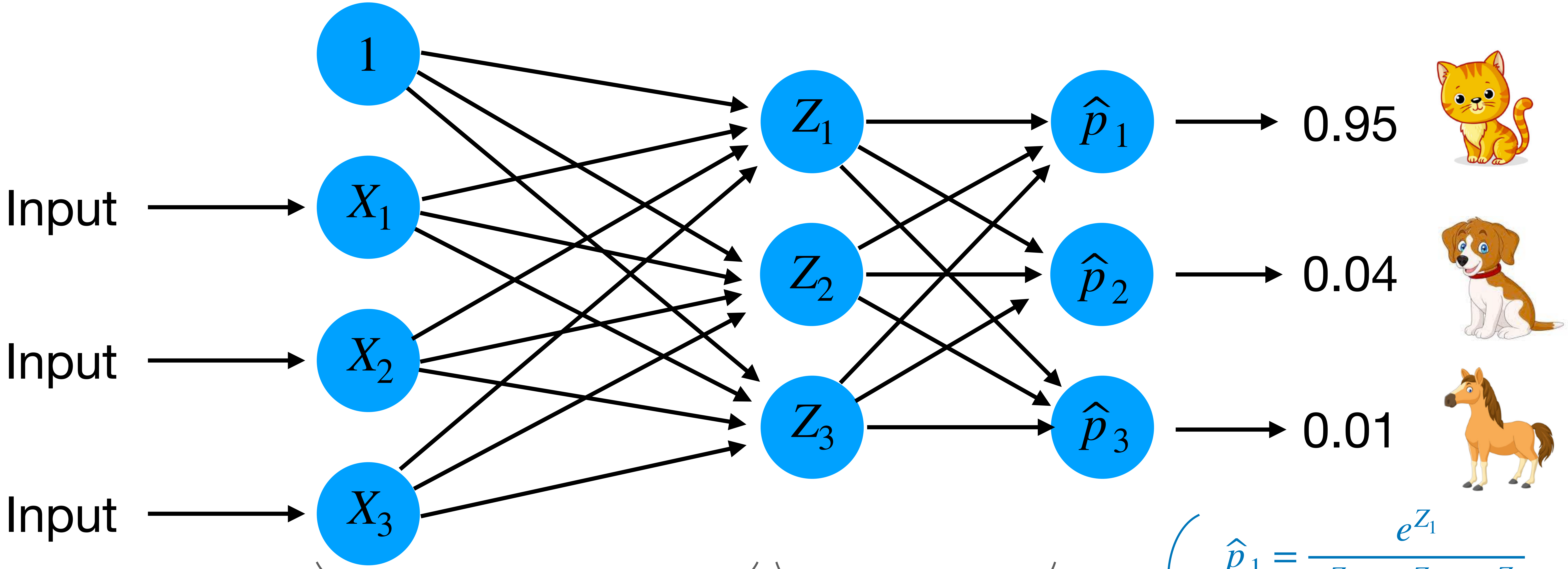

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~~sigmoid activation~~ softmax activation

$$\hat{p}_1 = \frac{e^{Z_1}}{e^{Z_1} + e^{Z_2} + e^{Z_3}}$$

$$\hat{p}_2 = \frac{e^{Z_2}}{e^{Z_1} + e^{Z_2} + e^{Z_3}}$$

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The cross-entropy loss function

Suppose we have a true label Y and fitted probabilities $\widehat{p}_1, \widehat{p}_2, \widehat{p}_3$. Define

$$\text{cross-entropy loss } L(Y, \widehat{p}) = \begin{cases} -\log(\widehat{p}_1) & \text{if } Y = 1; \\ -\log(\widehat{p}_2) & \text{if } Y = 2; \\ -\log(\widehat{p}_3) & \text{if } Y = 3. \end{cases}$$

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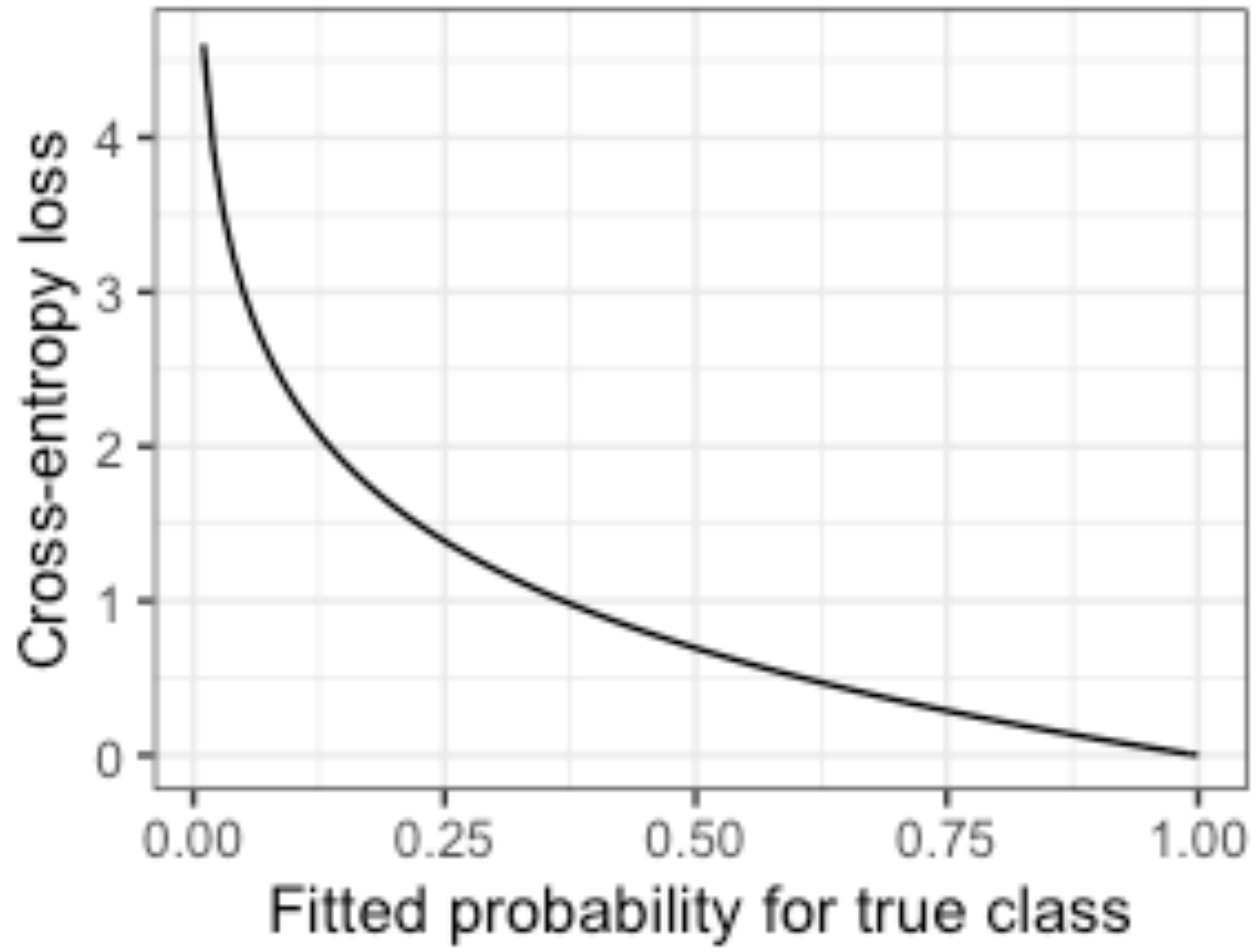

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Greater probability attached to true class
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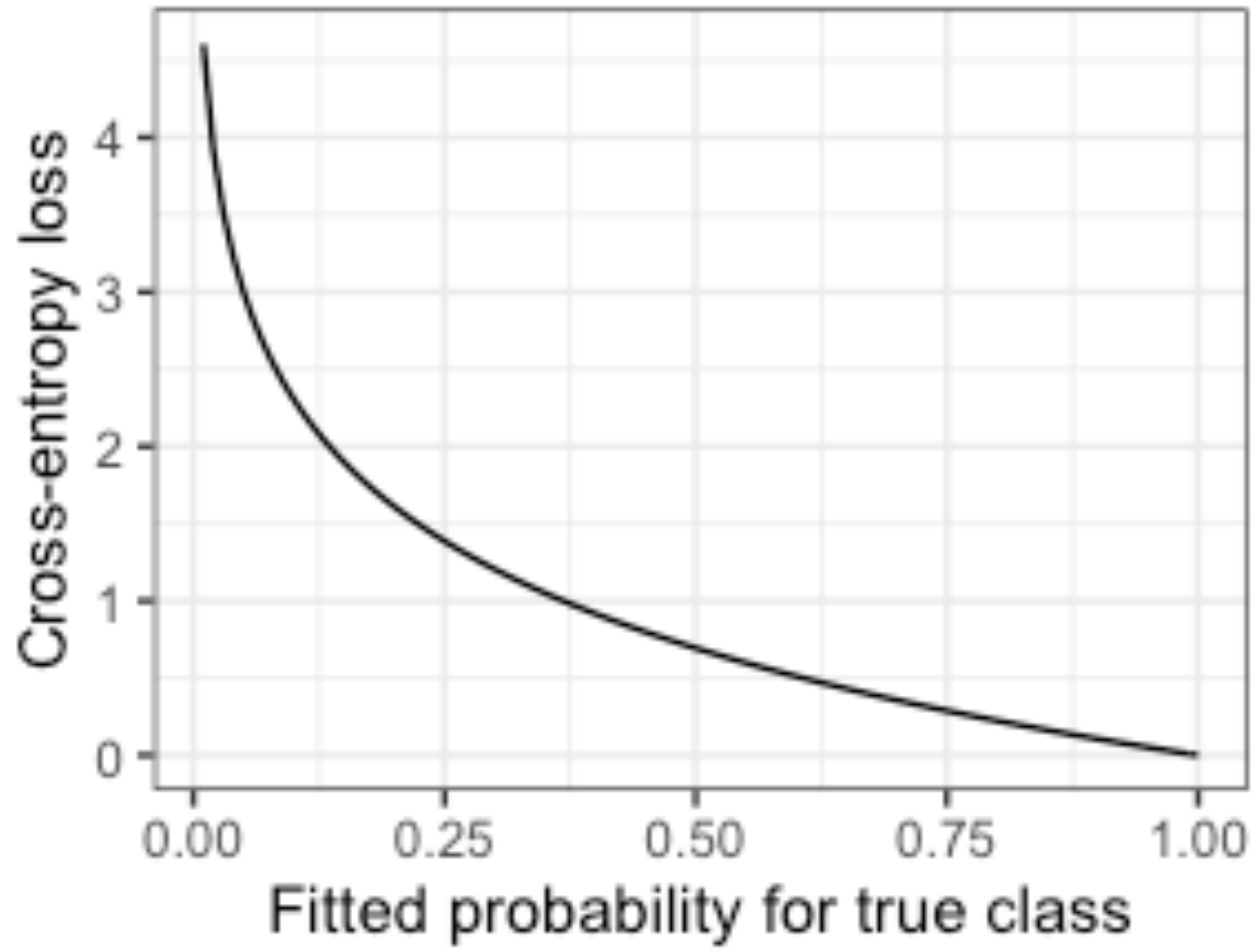

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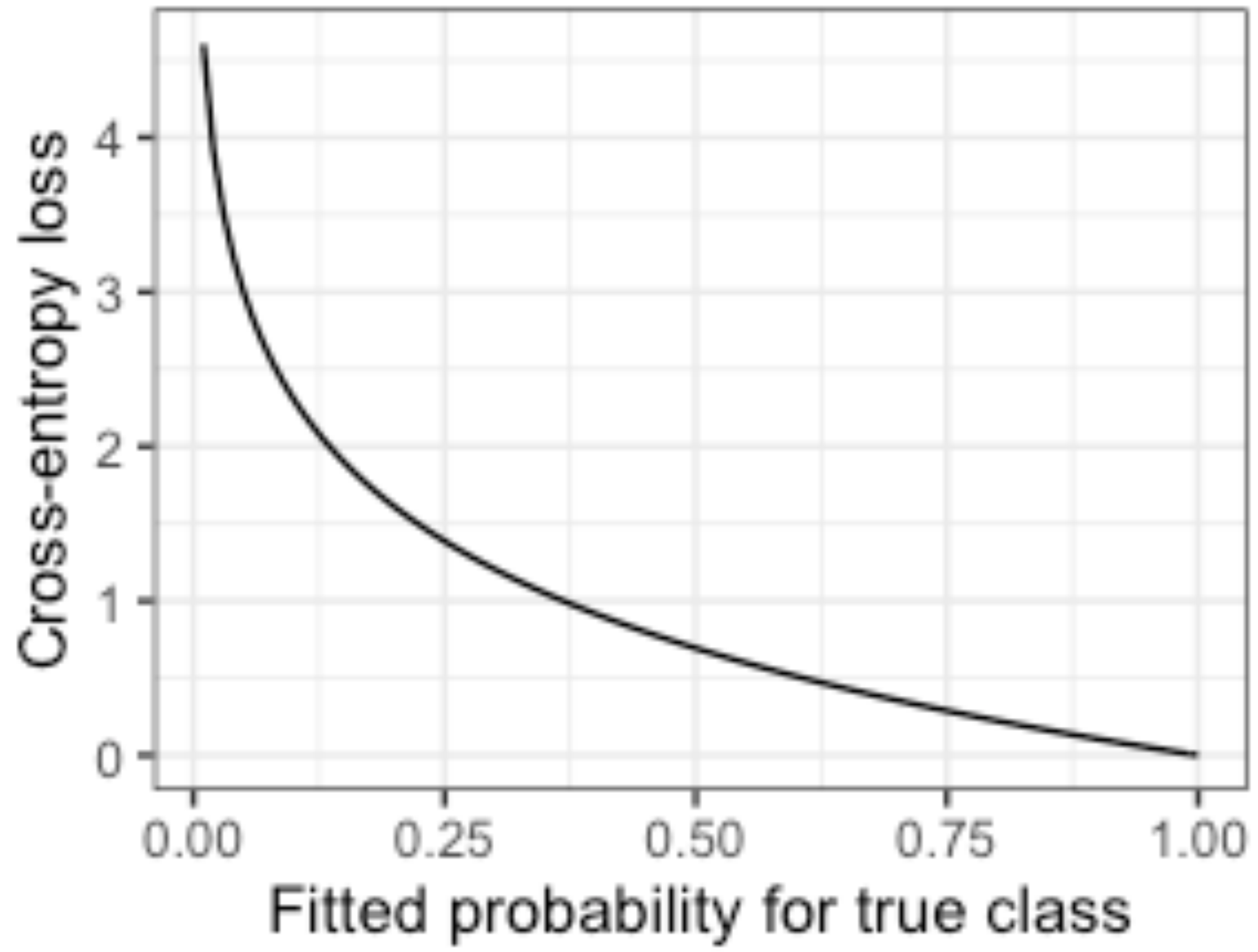
The cross-entropy loss generalizes the negative logarithm of the logistic likelihood.



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Find member of this class that best fits the training data, as measured by the loss function L of predictions given true responses, possibly regularized:

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For example, ridge regression has

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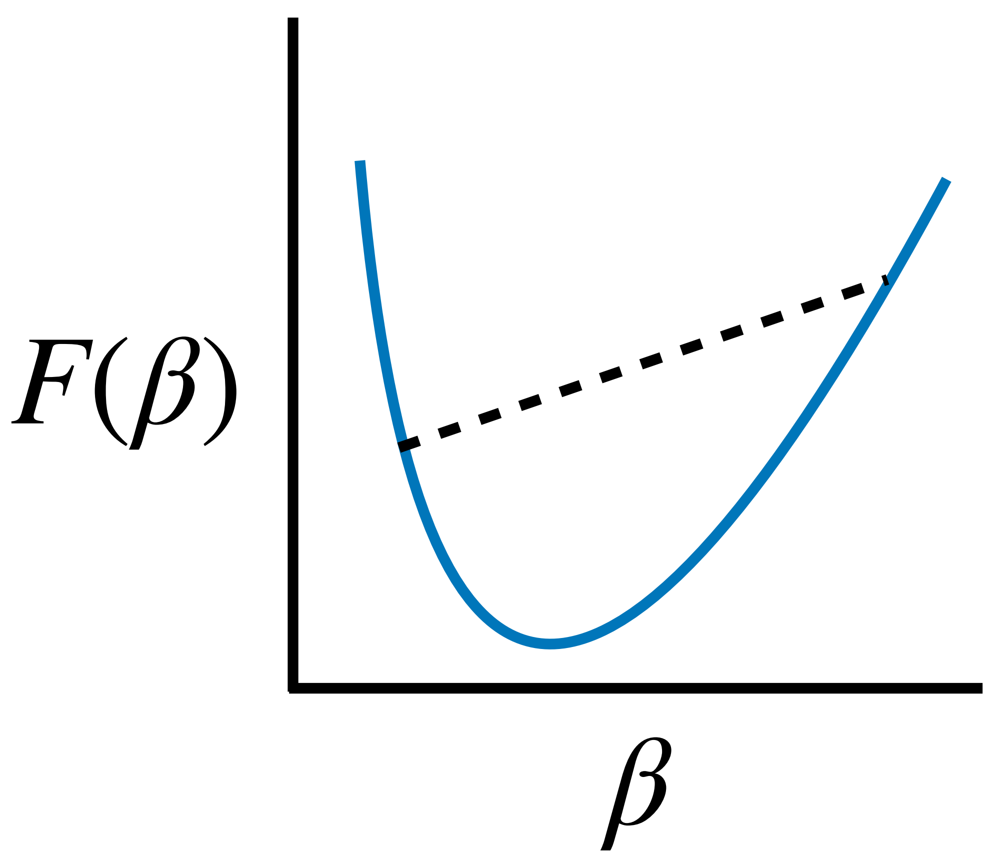
Training predictive models = solving optimization problems.

Convexity: A crucial property of F

The hardness of the optimization problem $\arg\min \; F(\beta)$ depends crucially on whether the objective function F is convex, or “bowl-shaped.”

Convex



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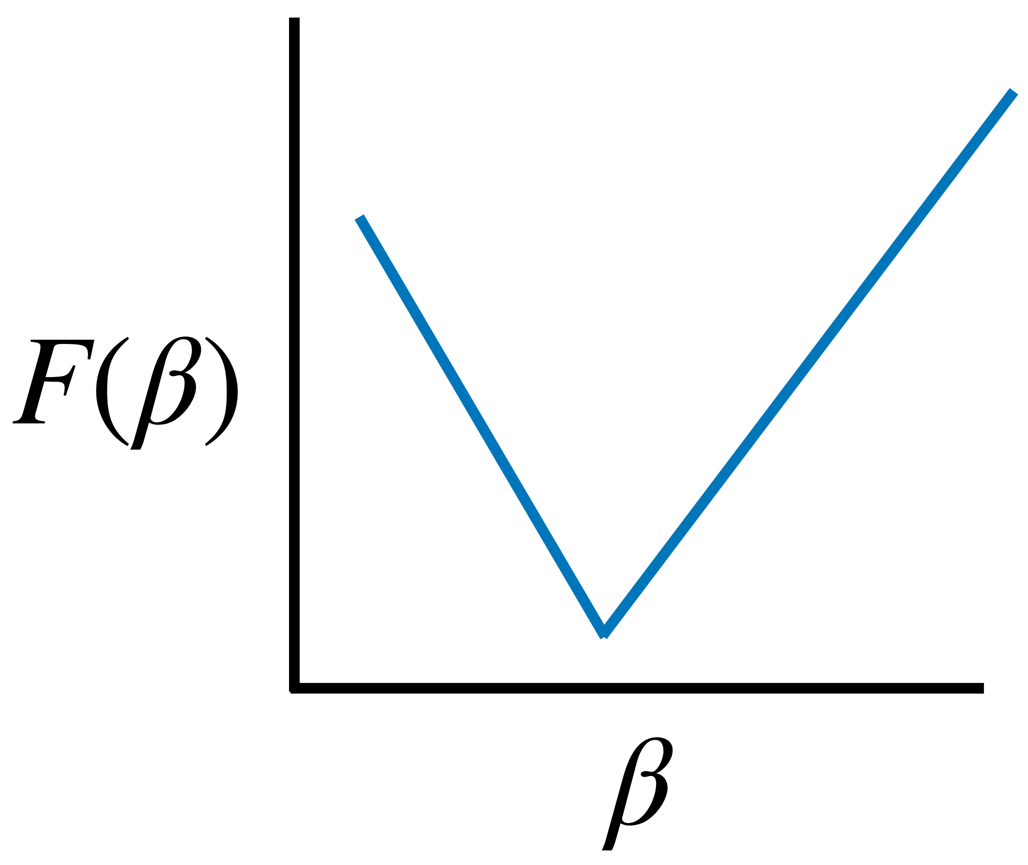
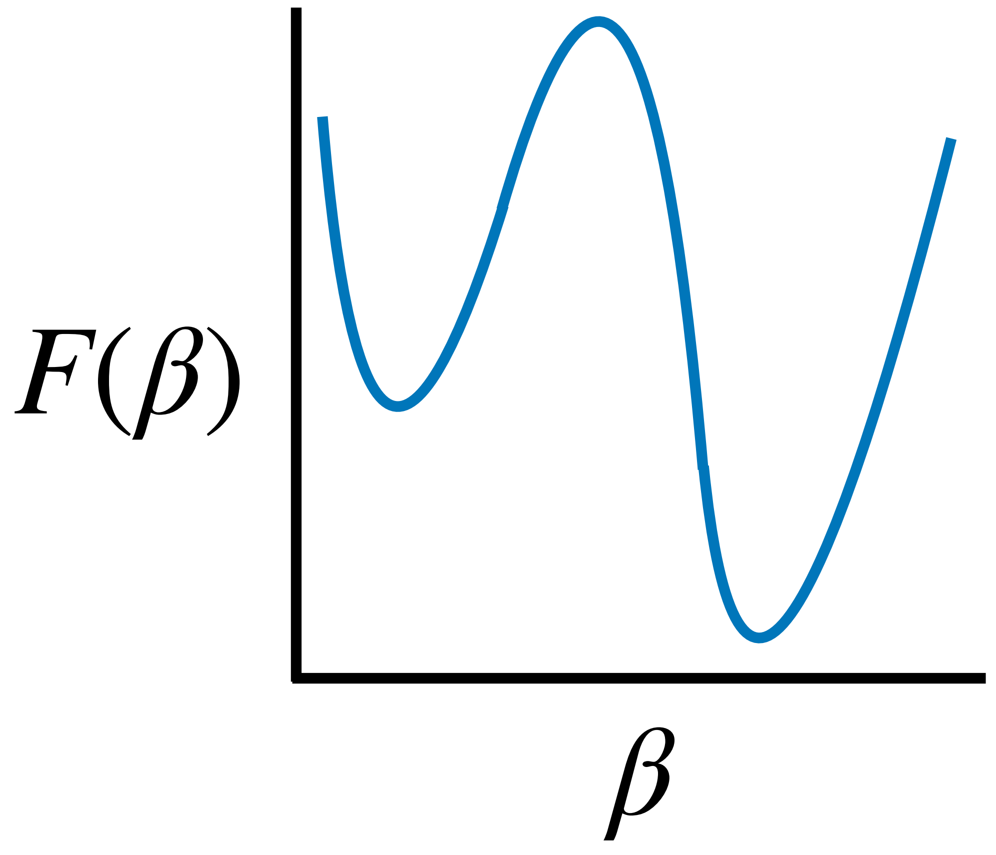


Non-convex



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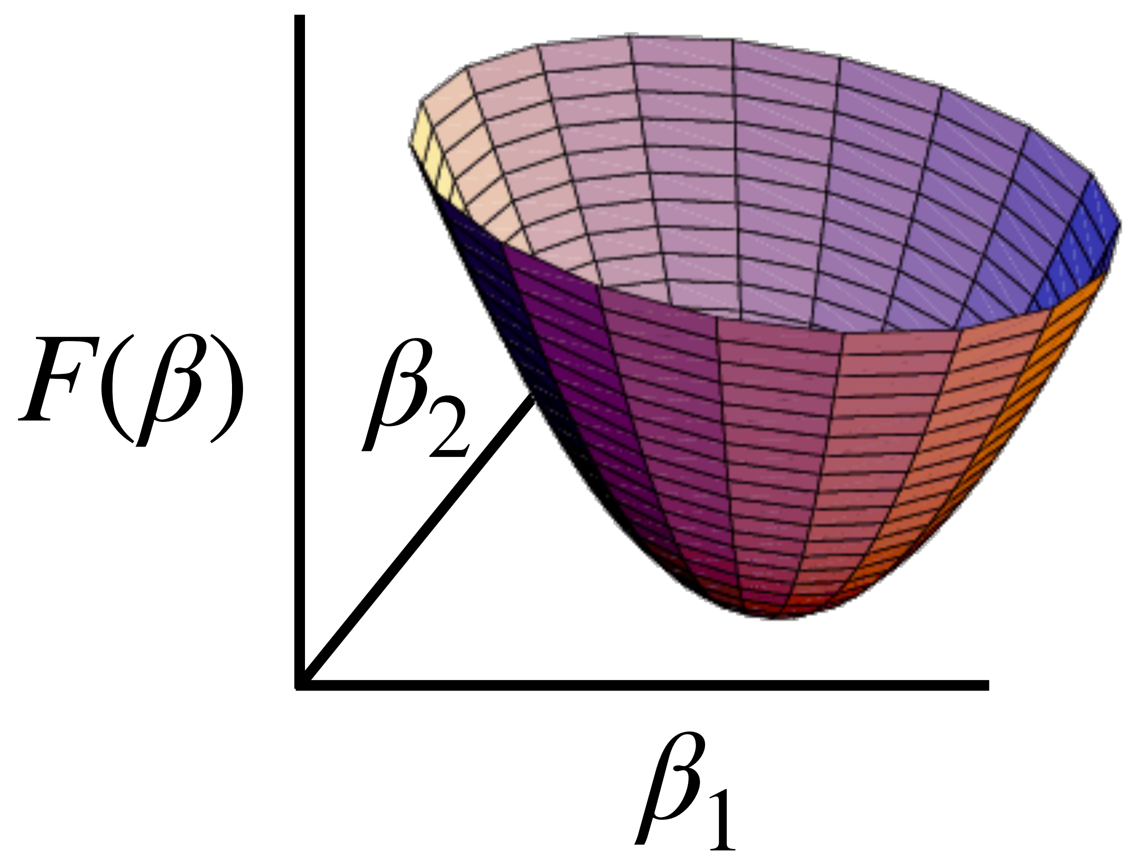

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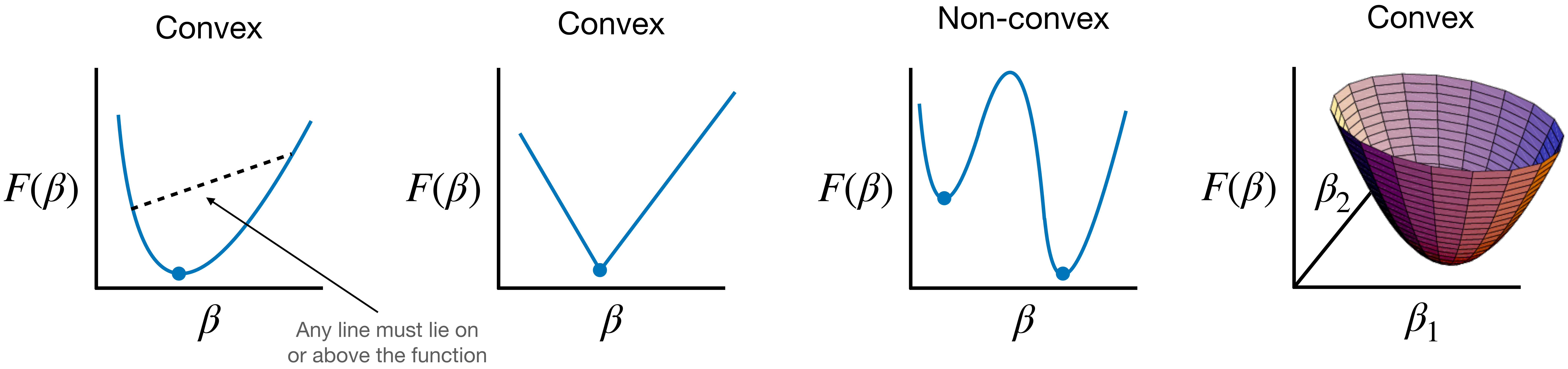

For convex functions, any local minimum must also be a global minimum.

It is much easier to find local minima than global minima.

Which methods have convex objectives?

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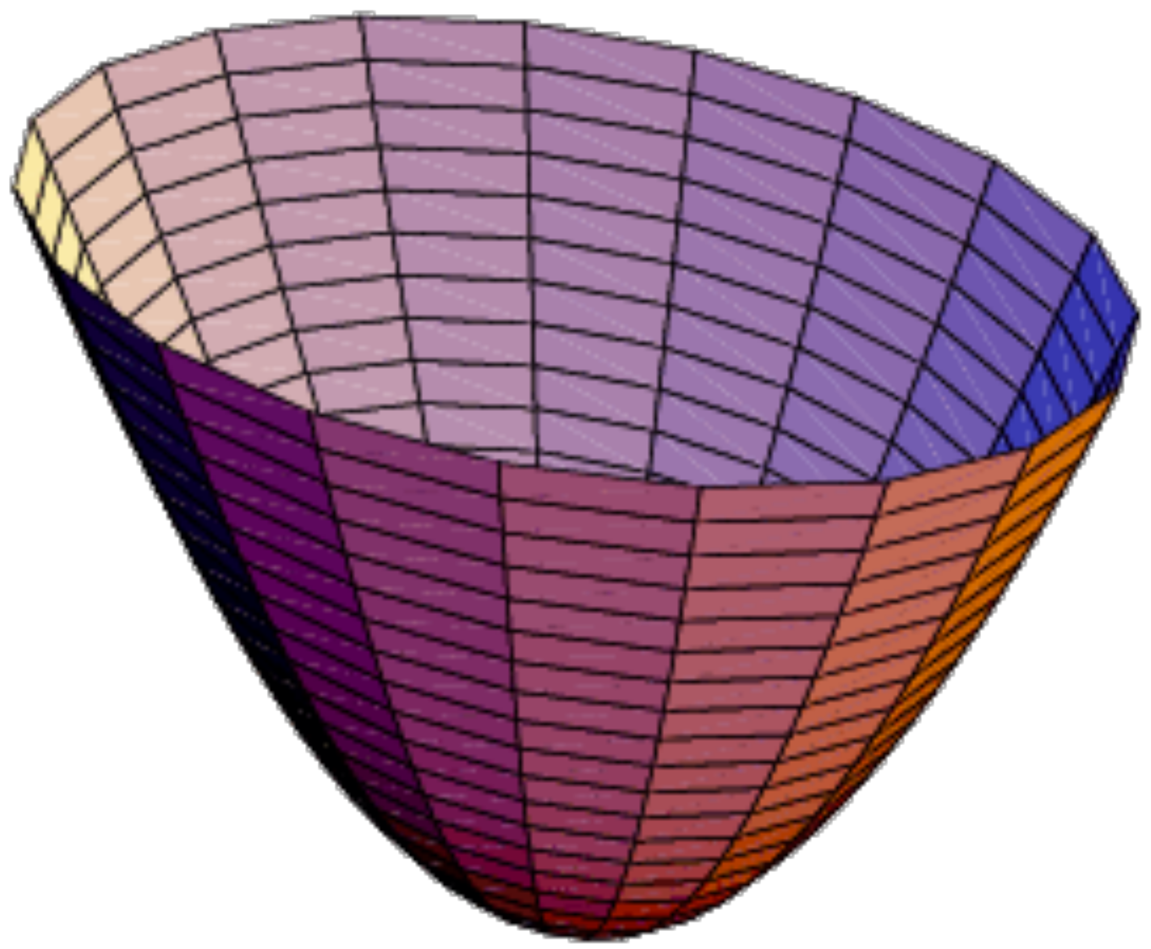
Convex

- Linear and logistic regression
- Linear and logistic regression with ridge or lasso penalties



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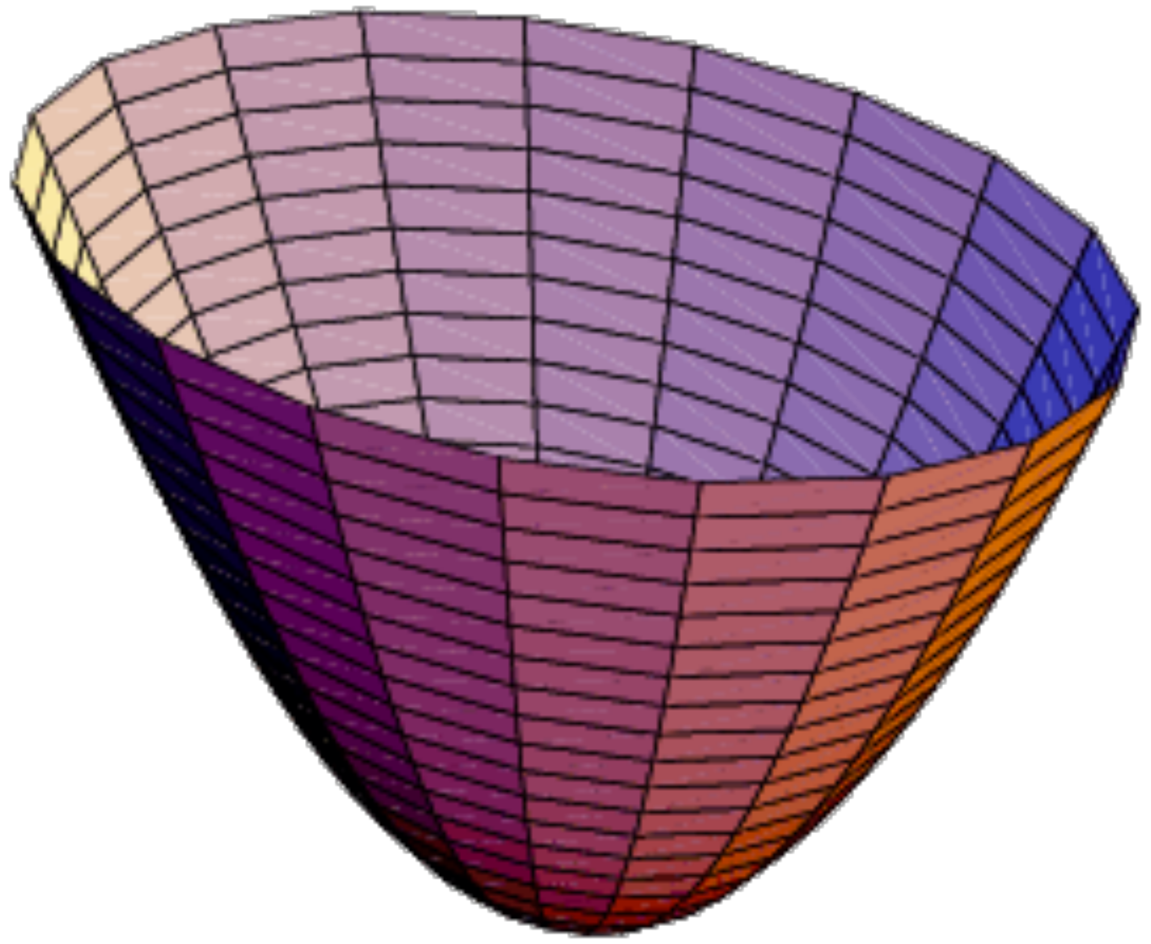
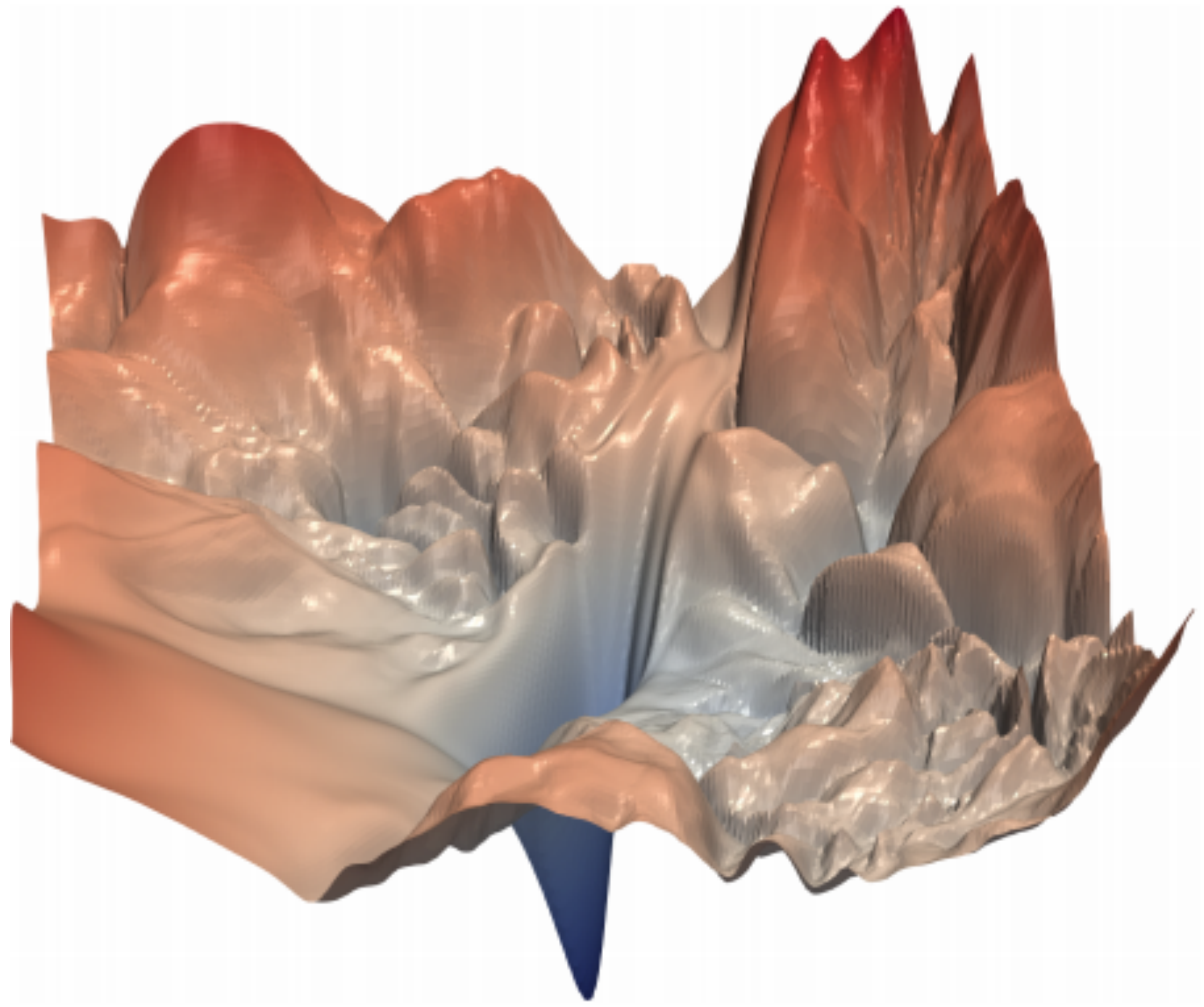


Not convex

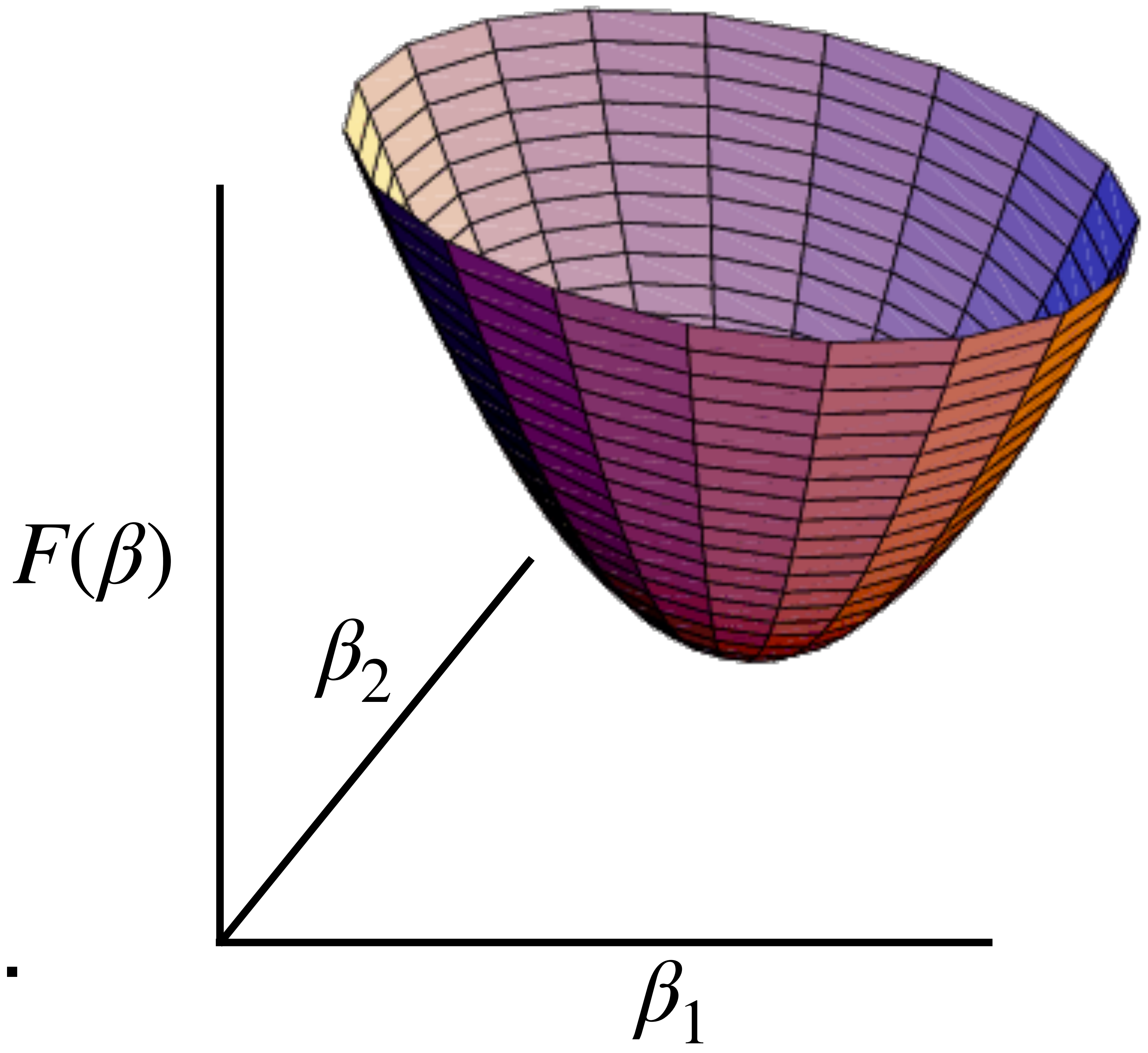
- Tree-based methods
- Neural networks



https://arxiv.org/abs/1712.09913

Gradient descent

1. Choose some initial value of β.
2. Evaluate the gradient $\nabla F(\beta)$ at that point; it is the direction in which F increases the fastest. The negative gradient is the direction in which F decreases the fastest.
3. Take small step in negative gradient direction: $\beta \leftarrow \beta - \gamma \nabla F(\beta)$; γ called the learning rate.
4. Repeat steps 2 and 3 until gradient is near zero.



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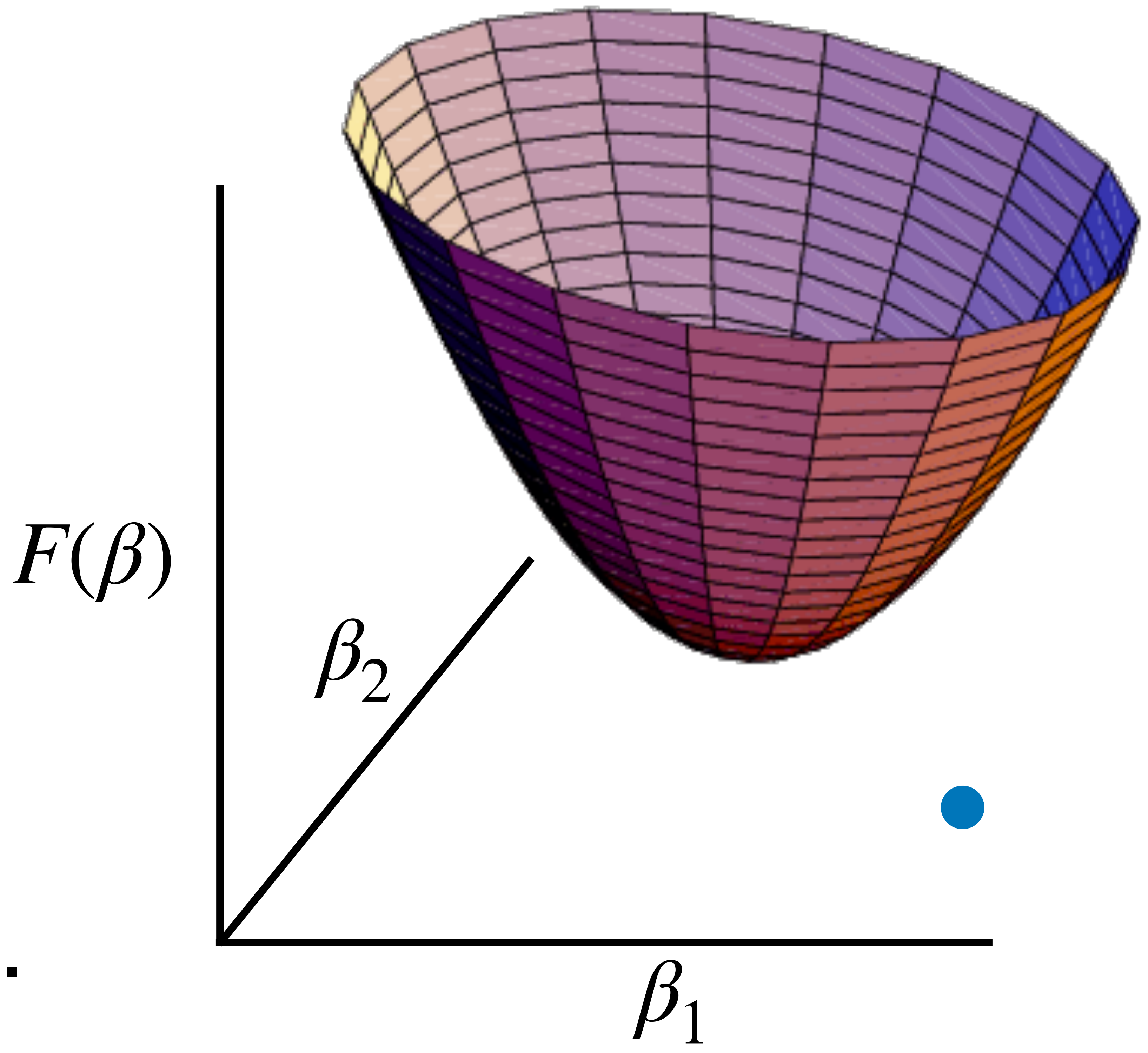

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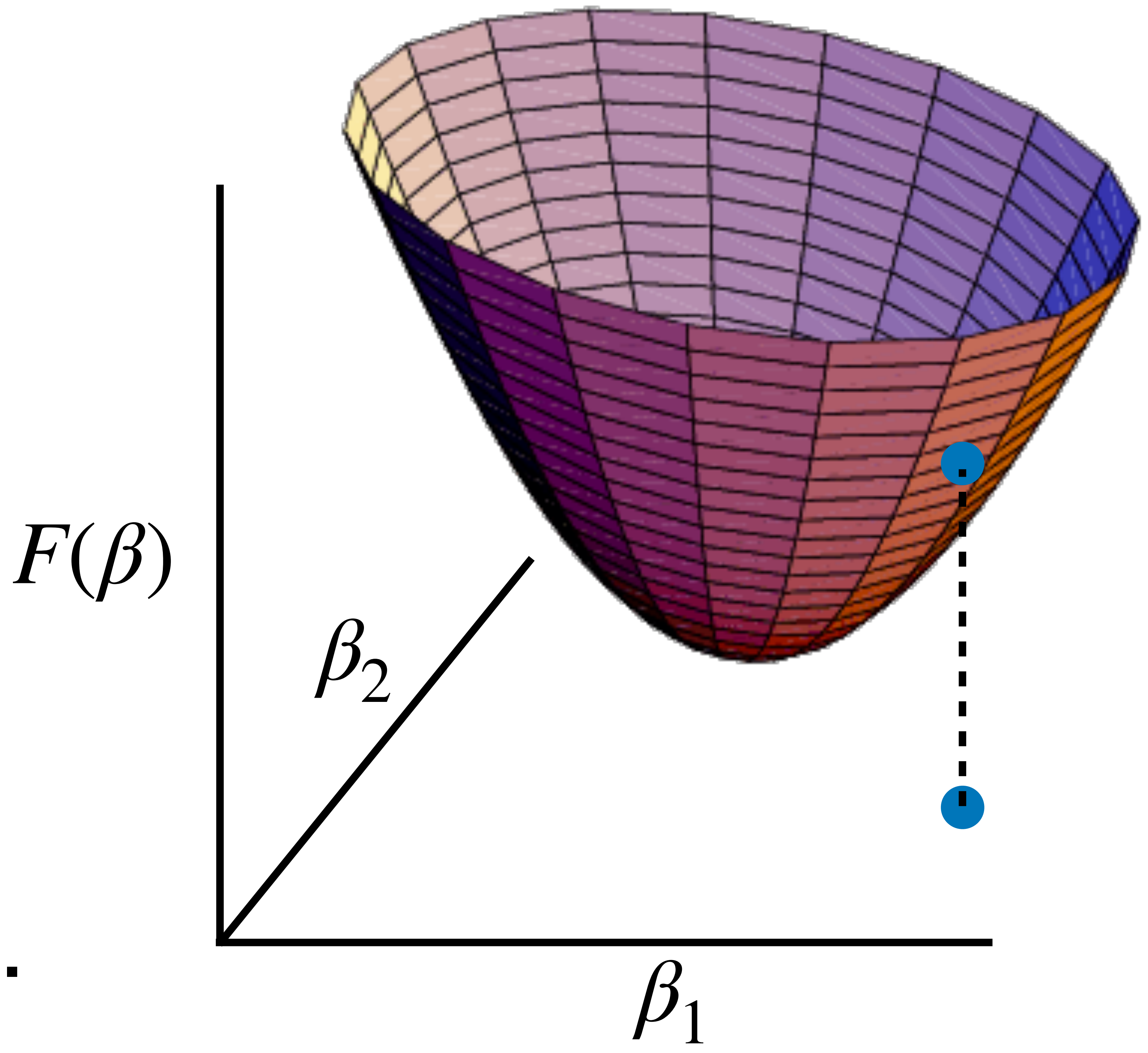

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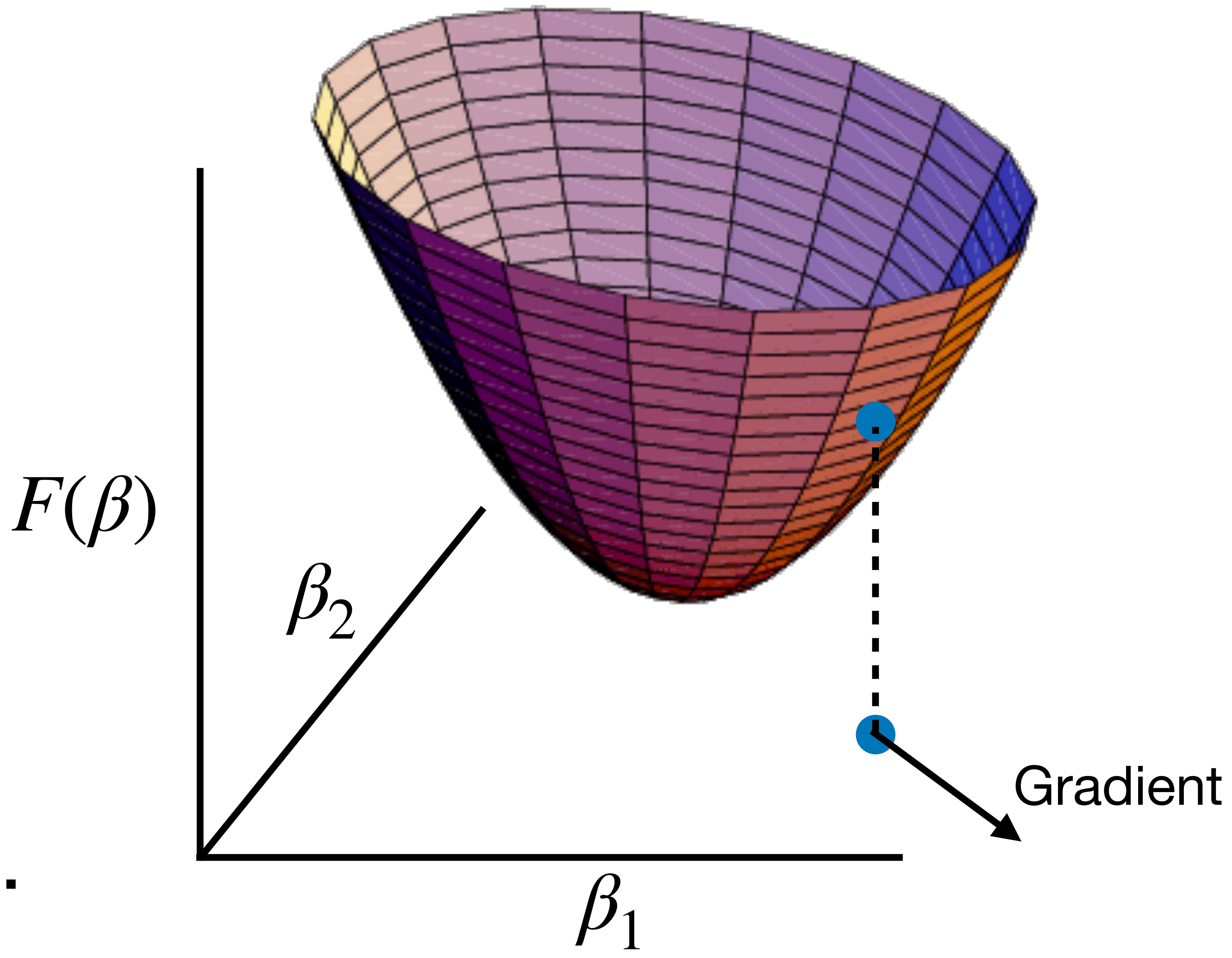

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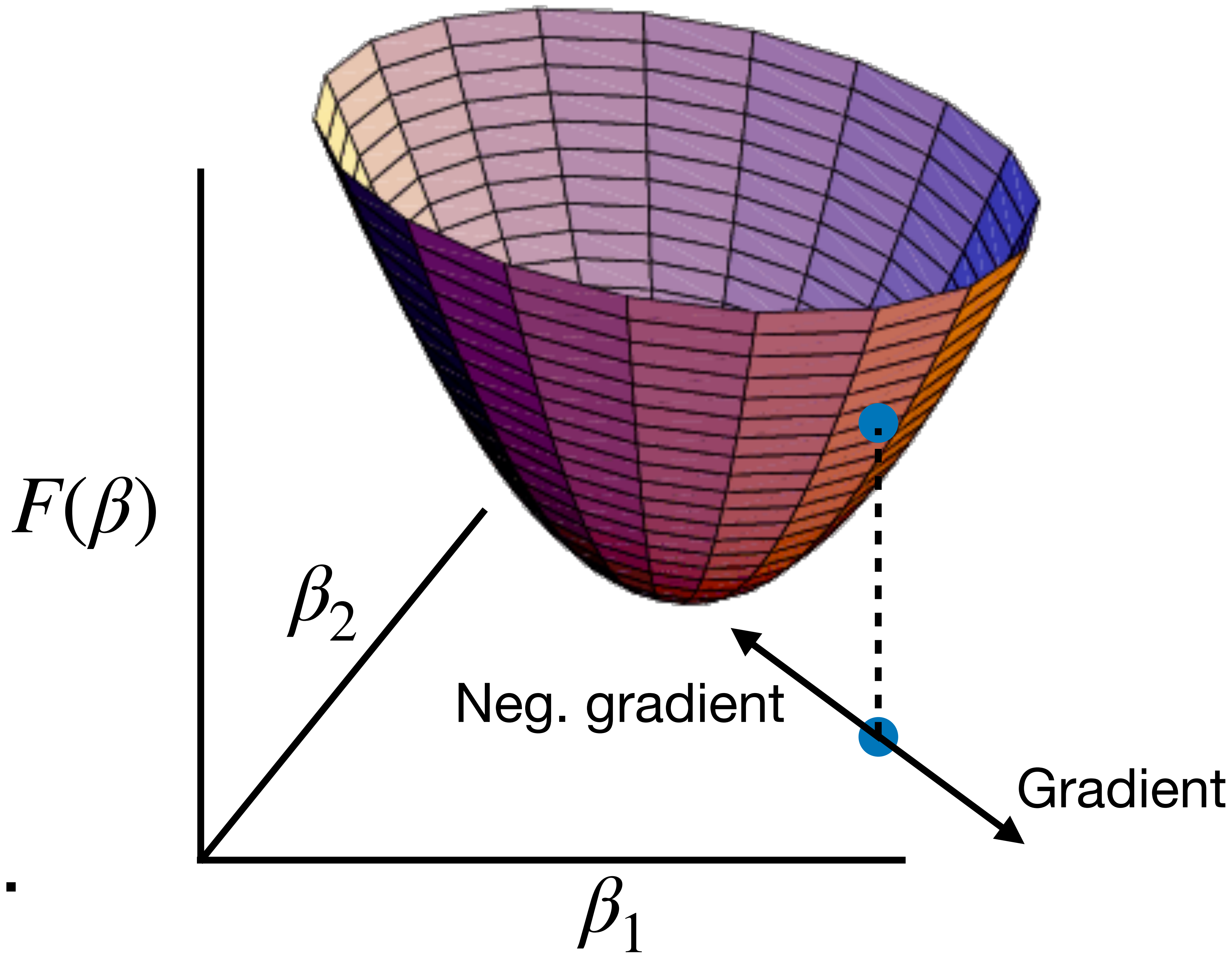

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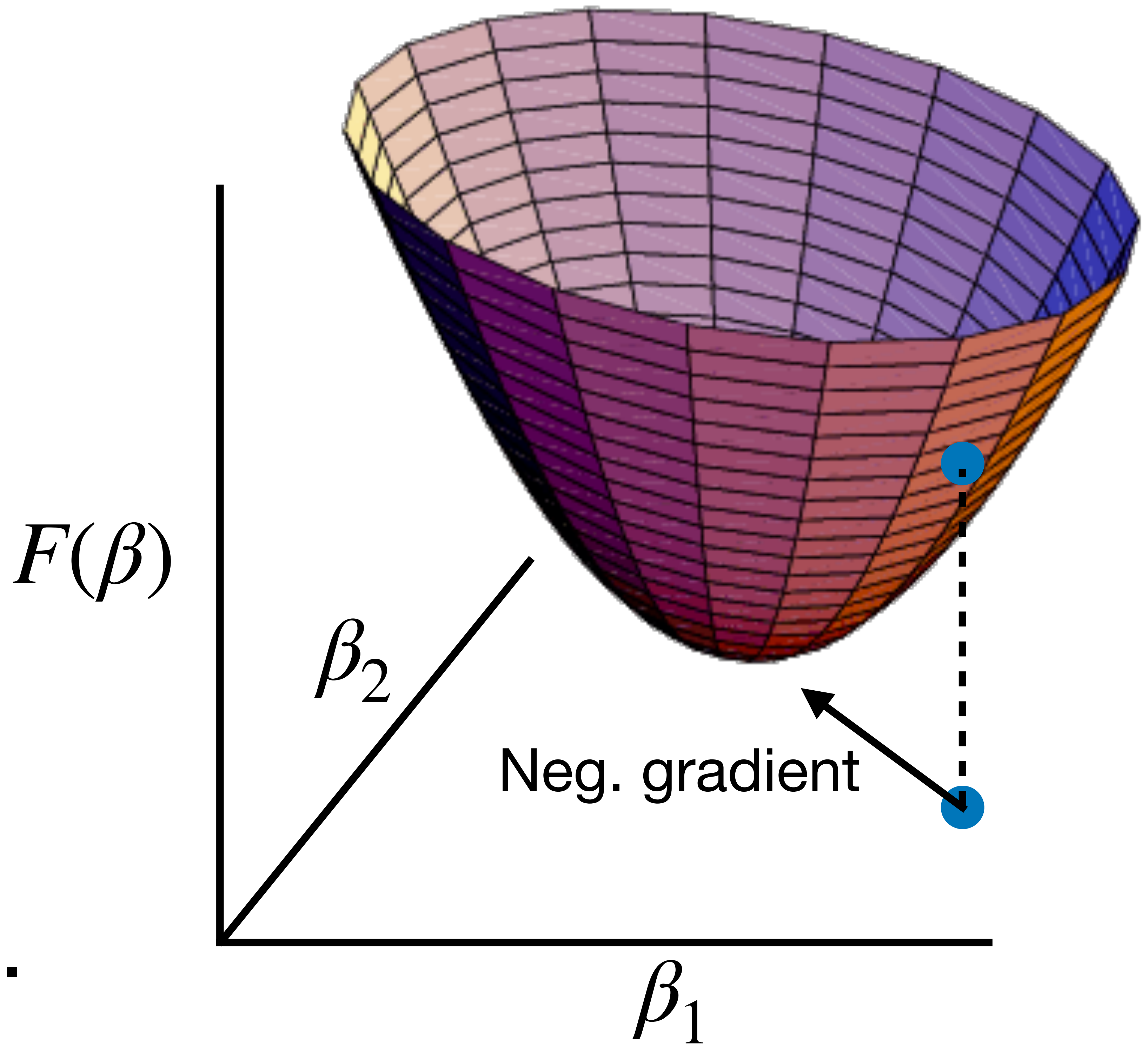

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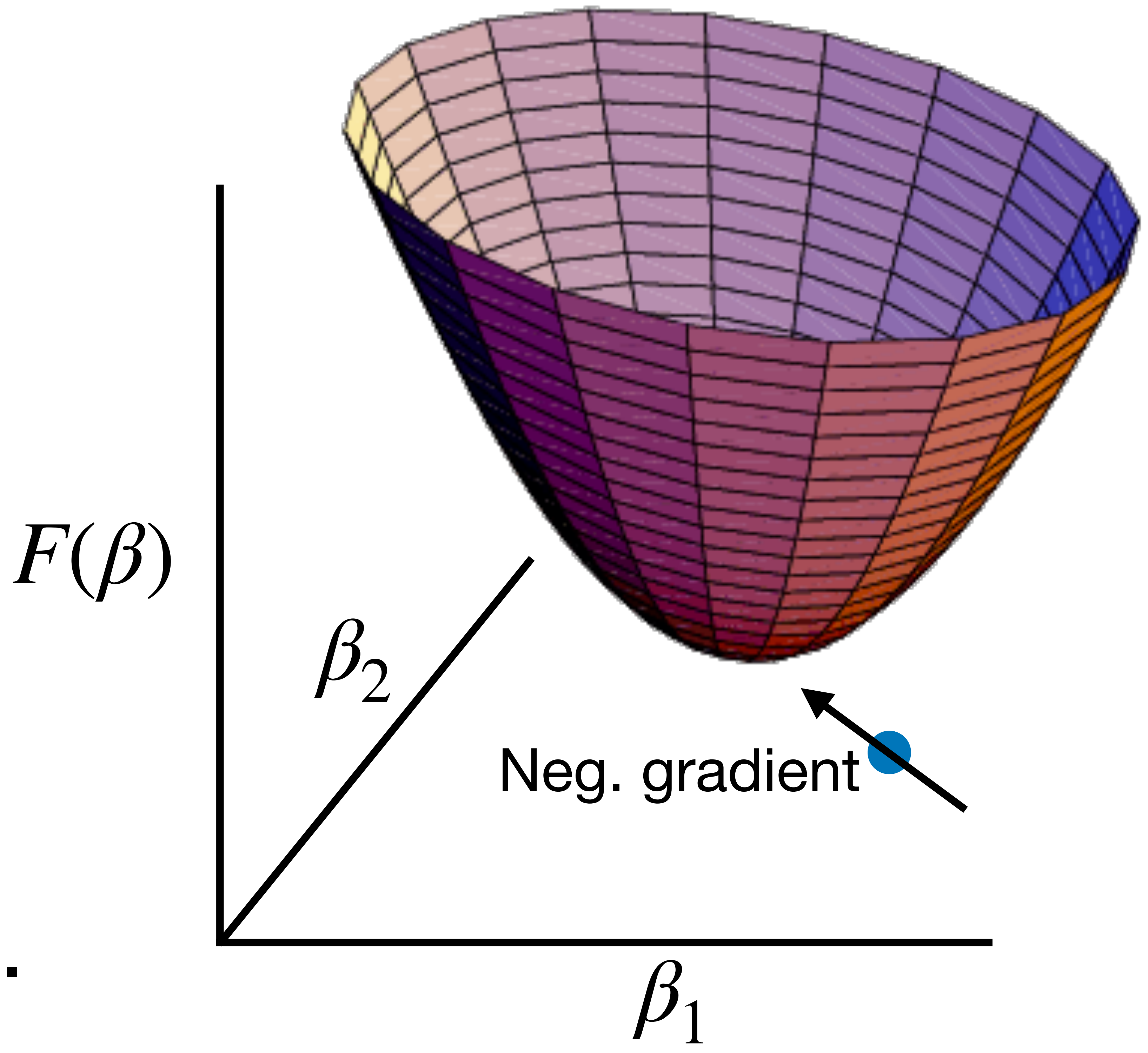

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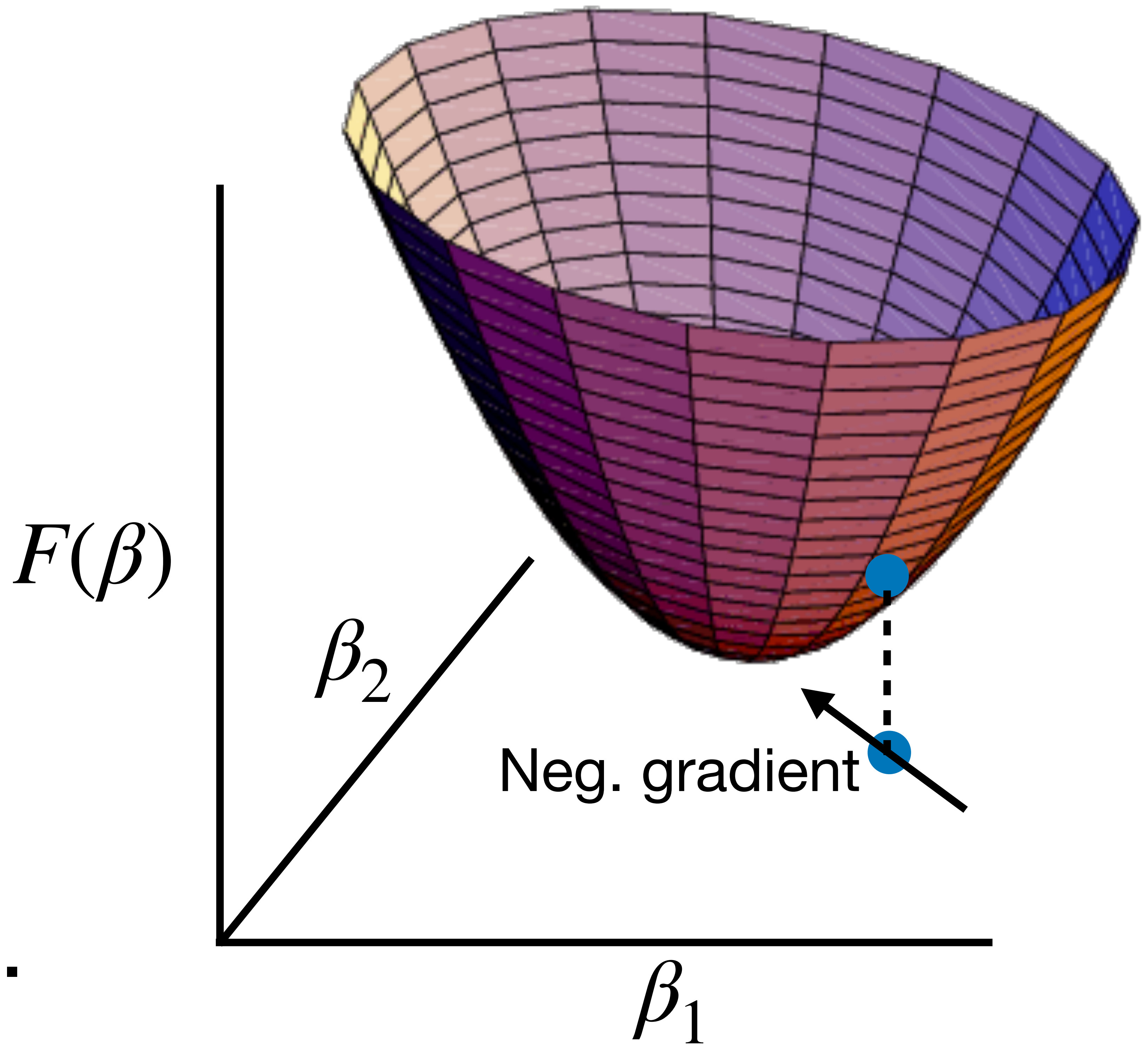

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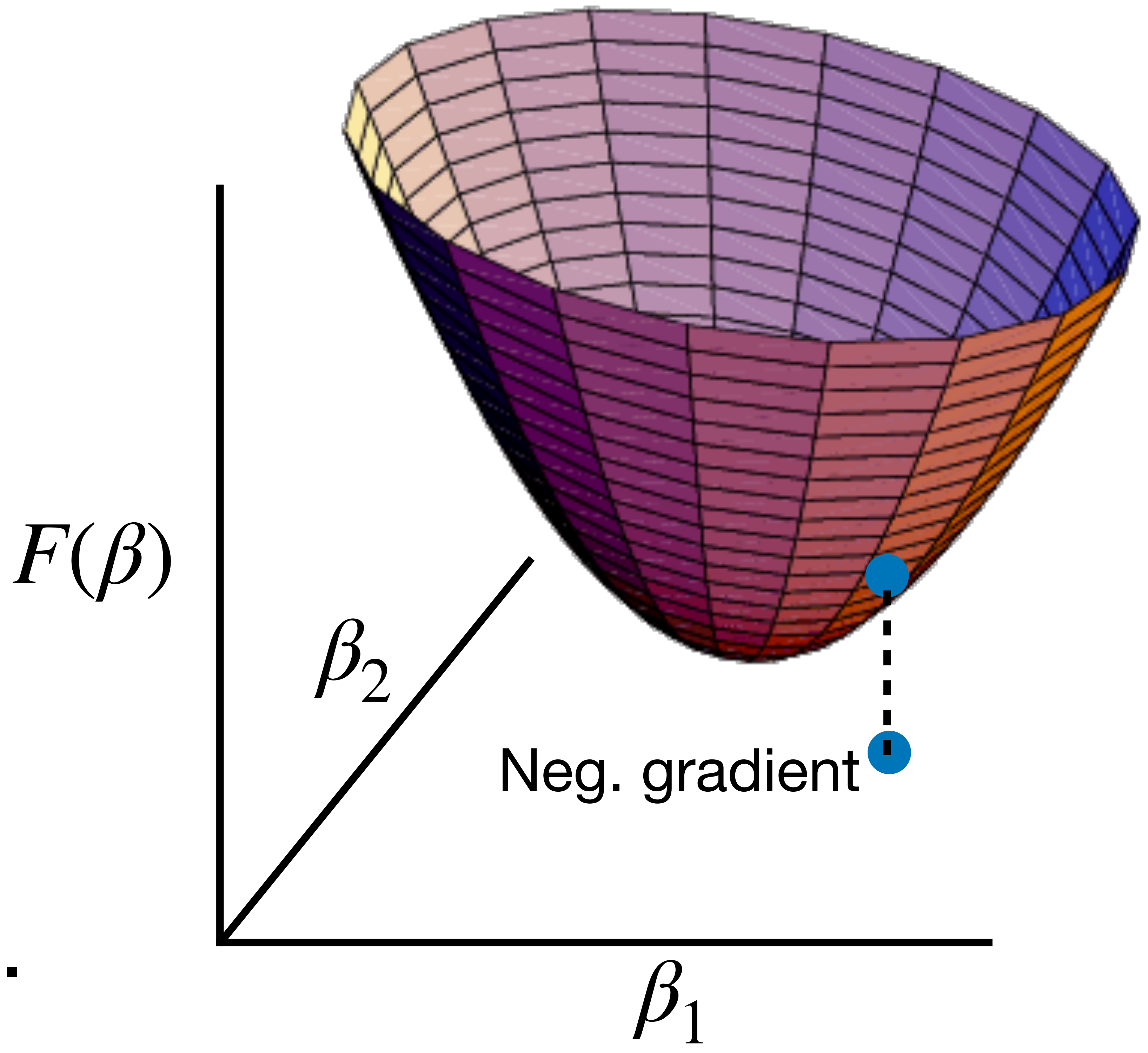

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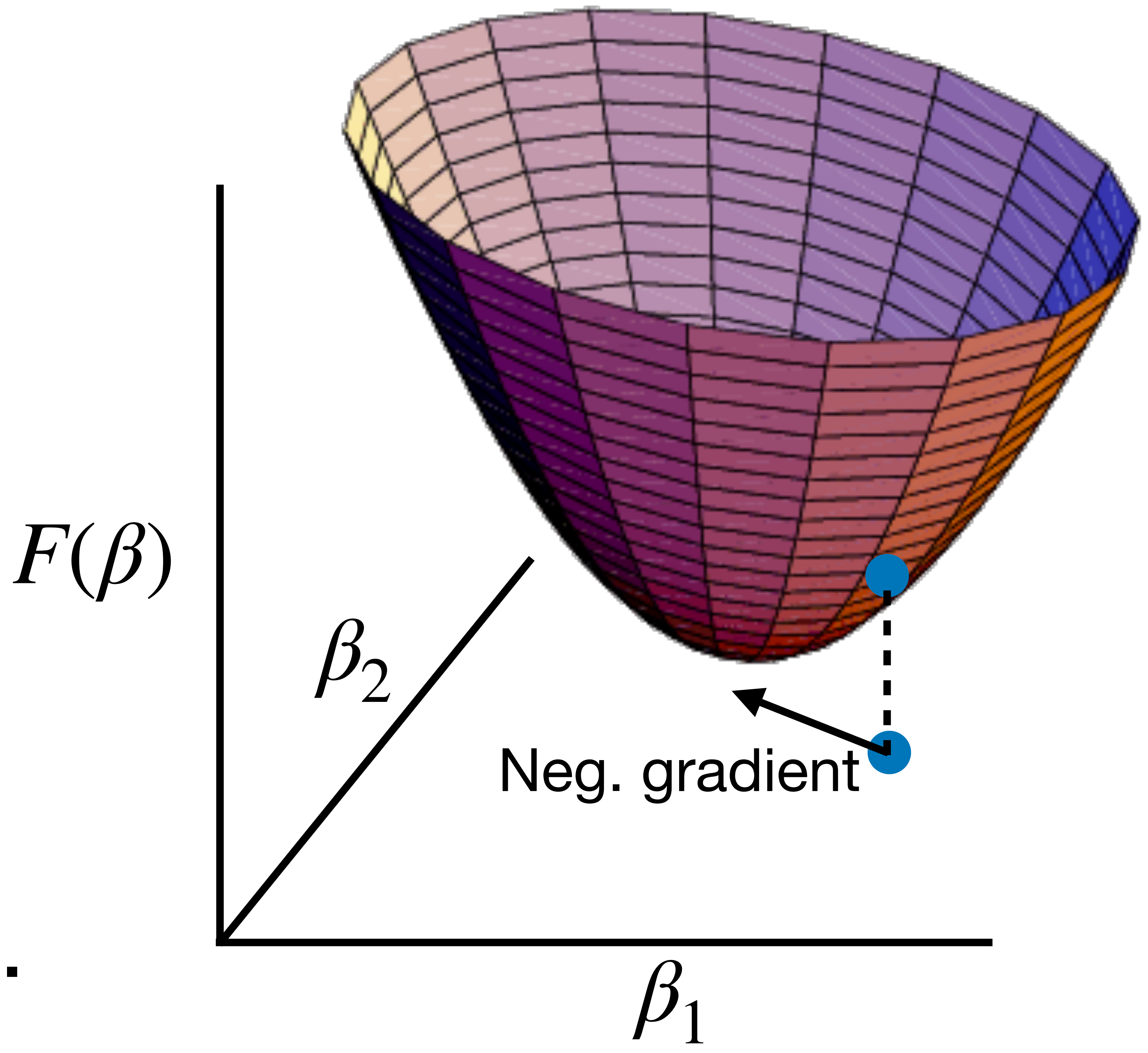

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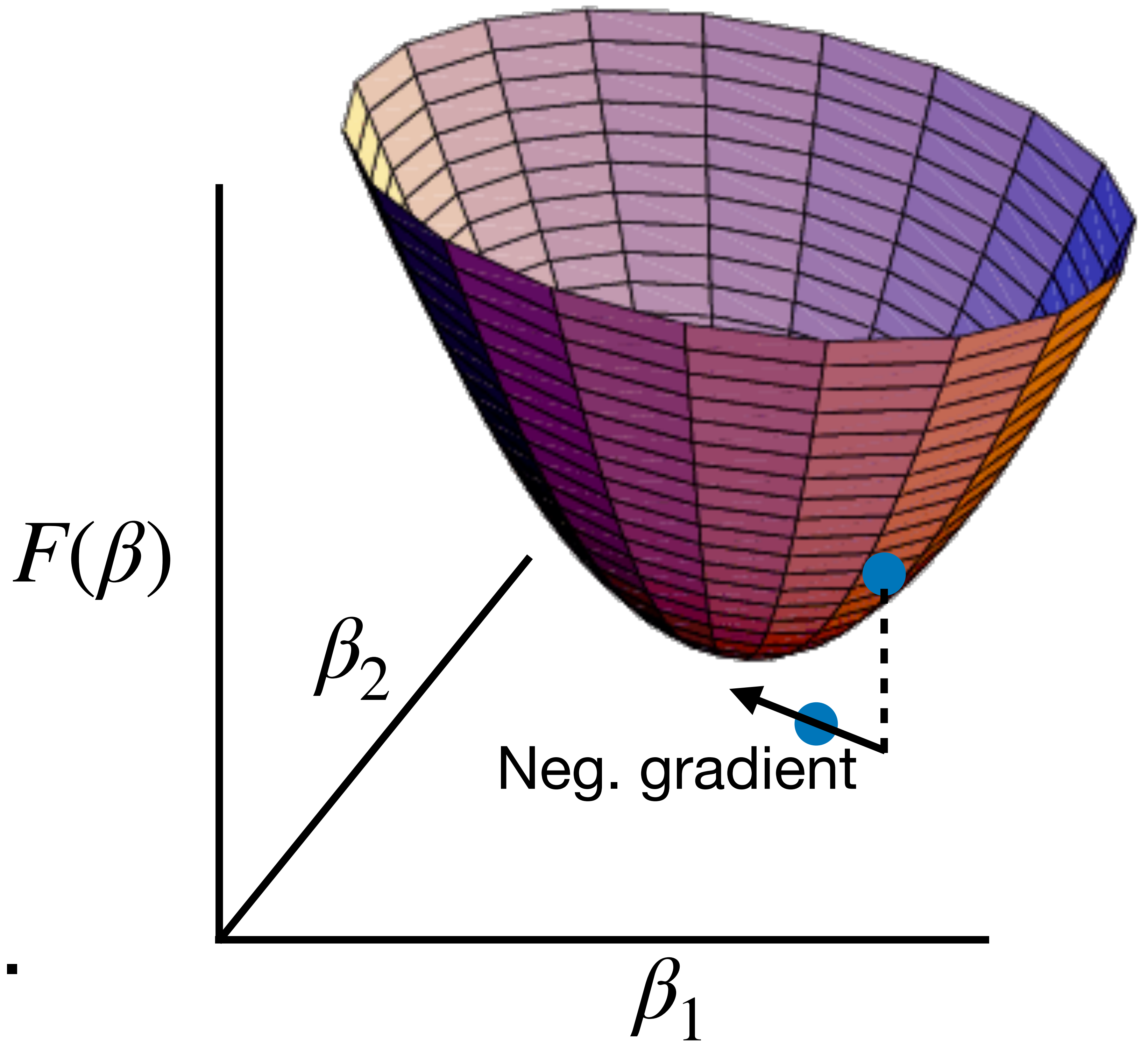

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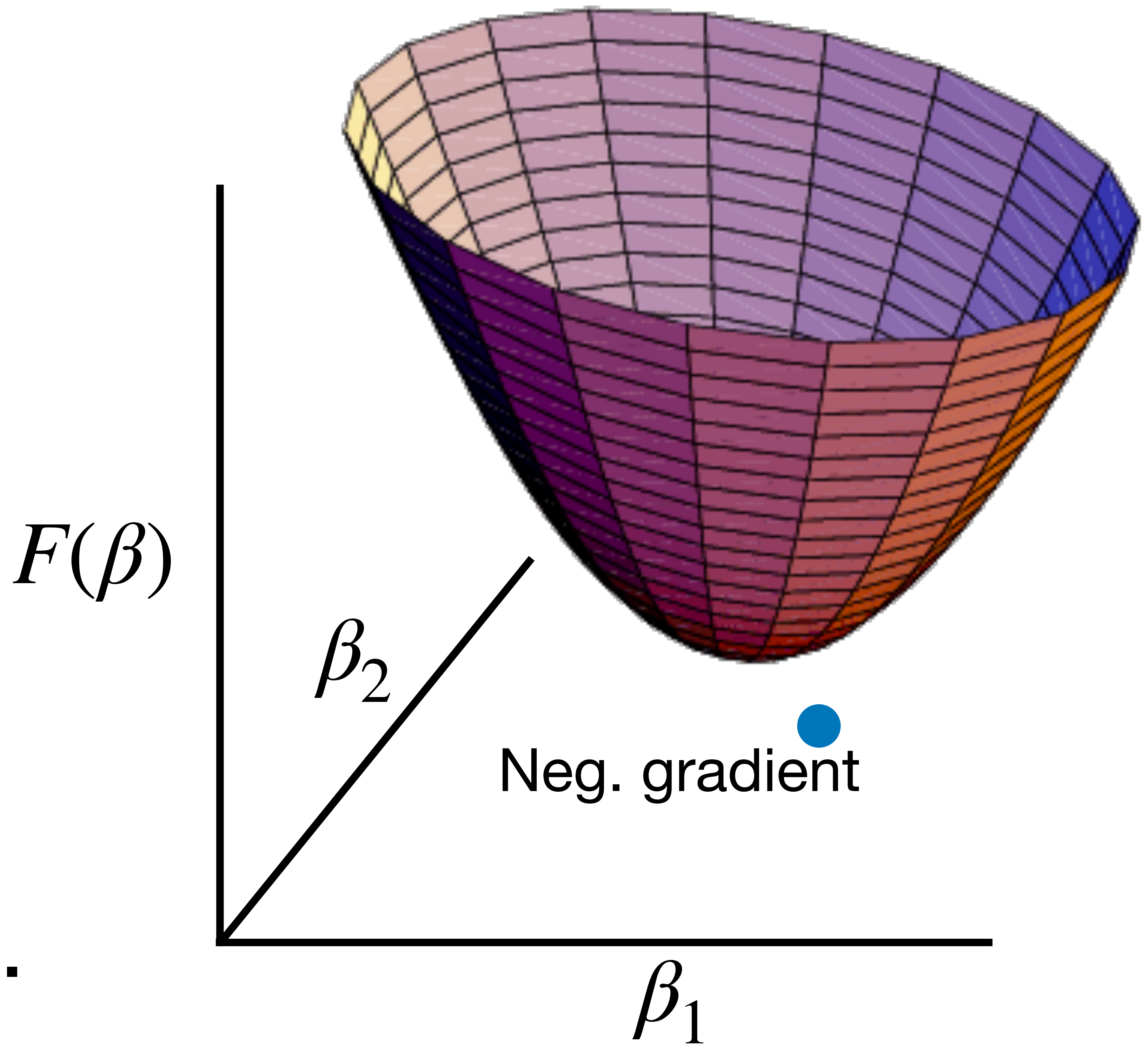

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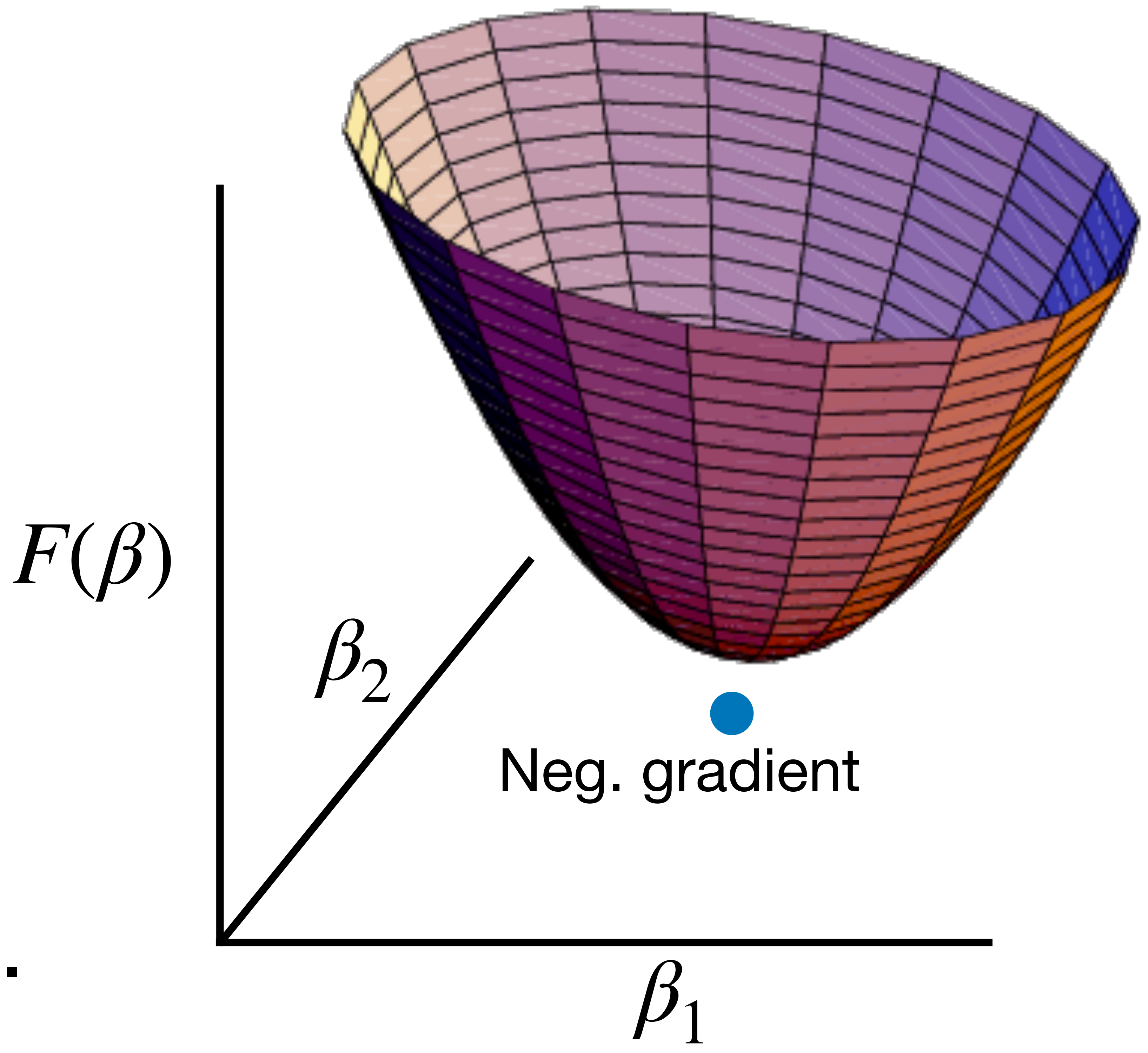

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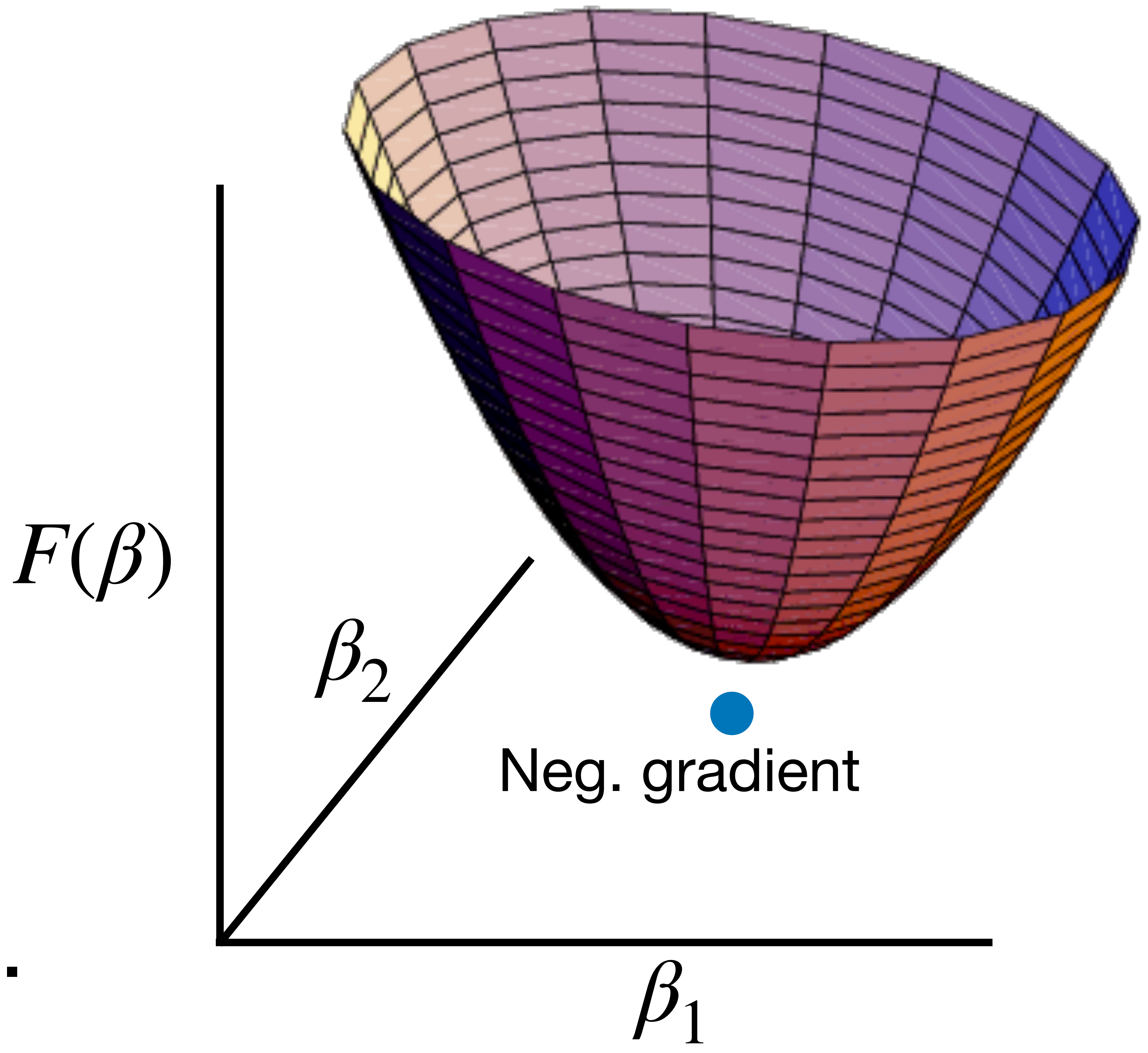

As long as the learning rate γ is not too large, gradient descent is guaranteed to converge to a global minimum regardless of initialization if F is convex.

Gradient descent for non-convex optimization

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Think about gradient descent as a ball rolling down a hill.

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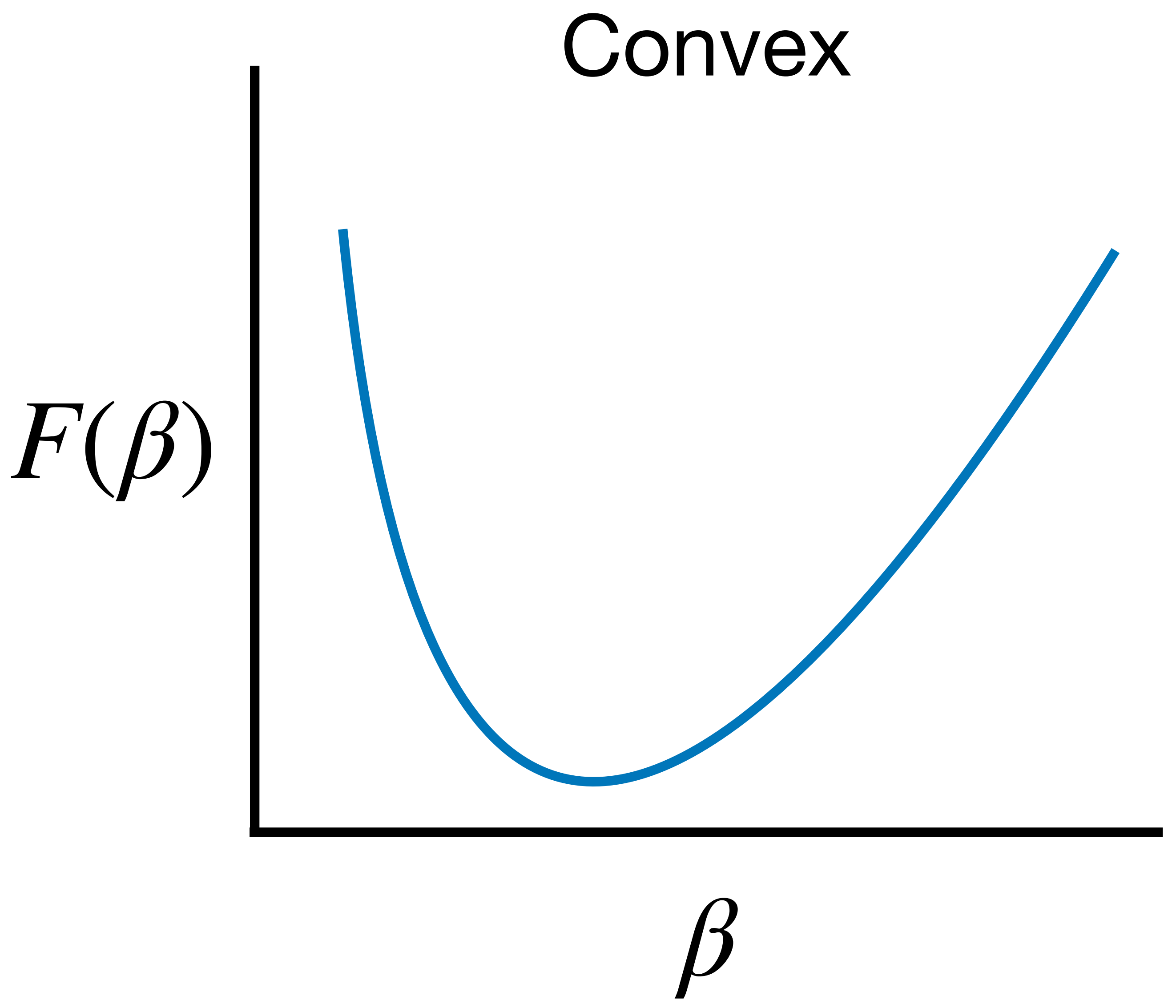

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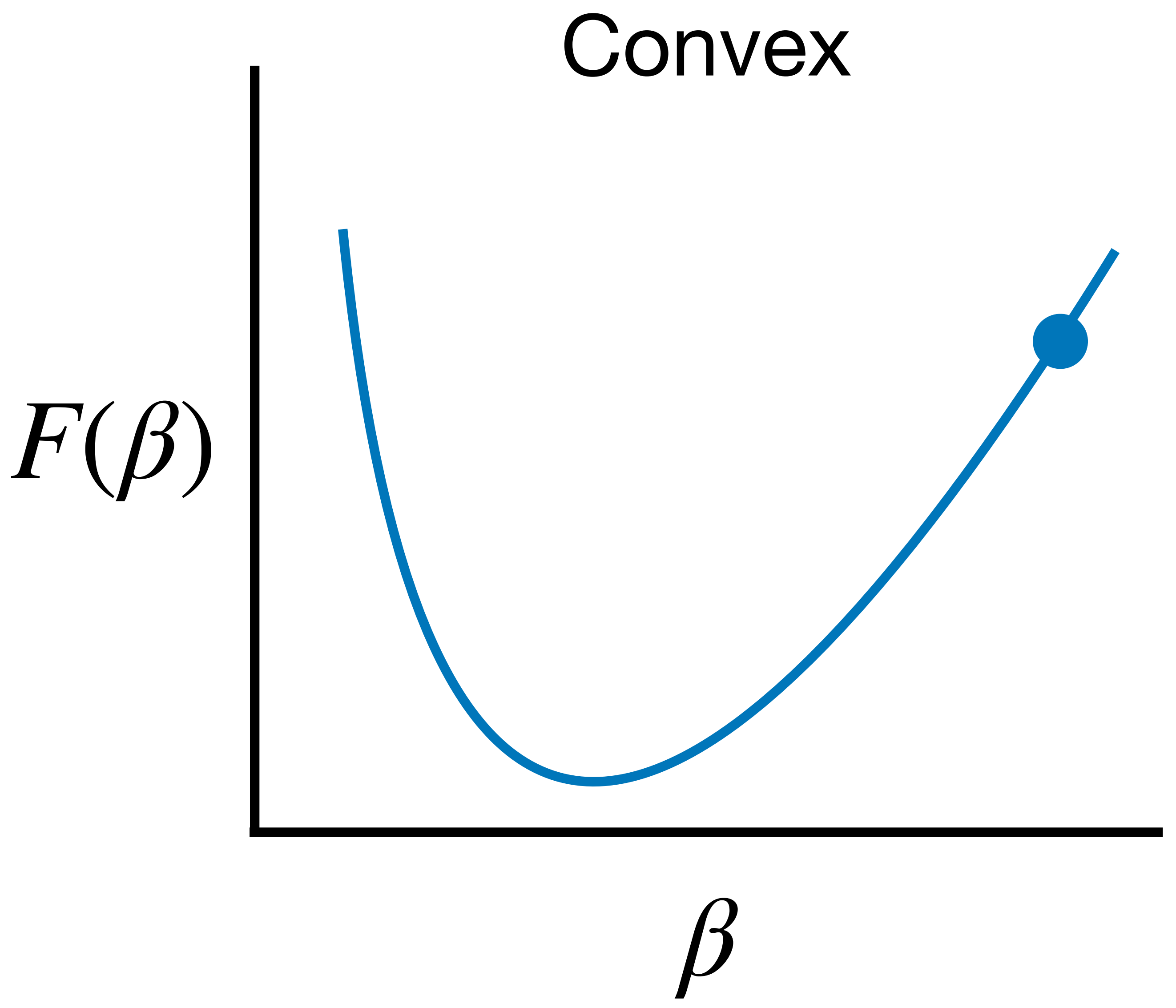

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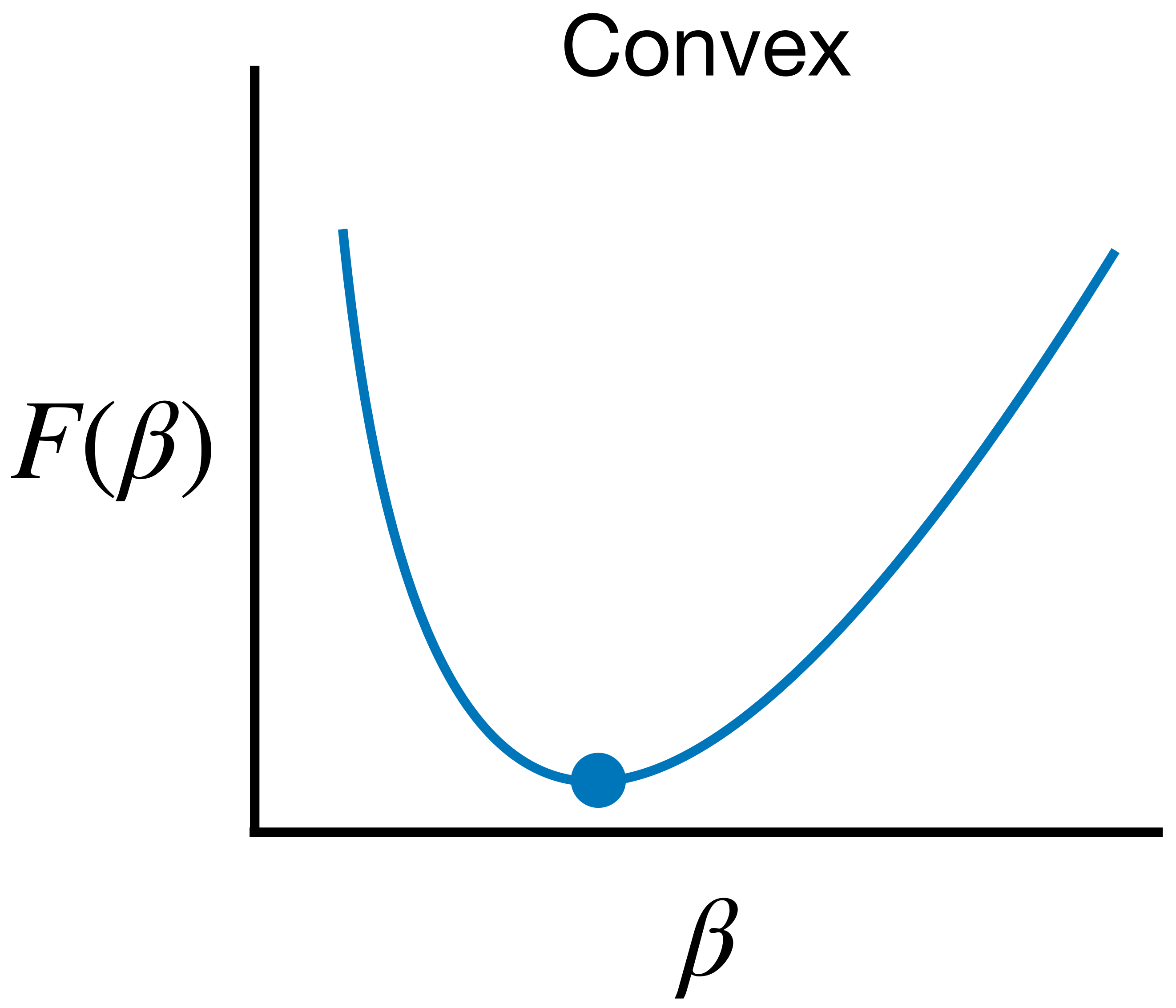

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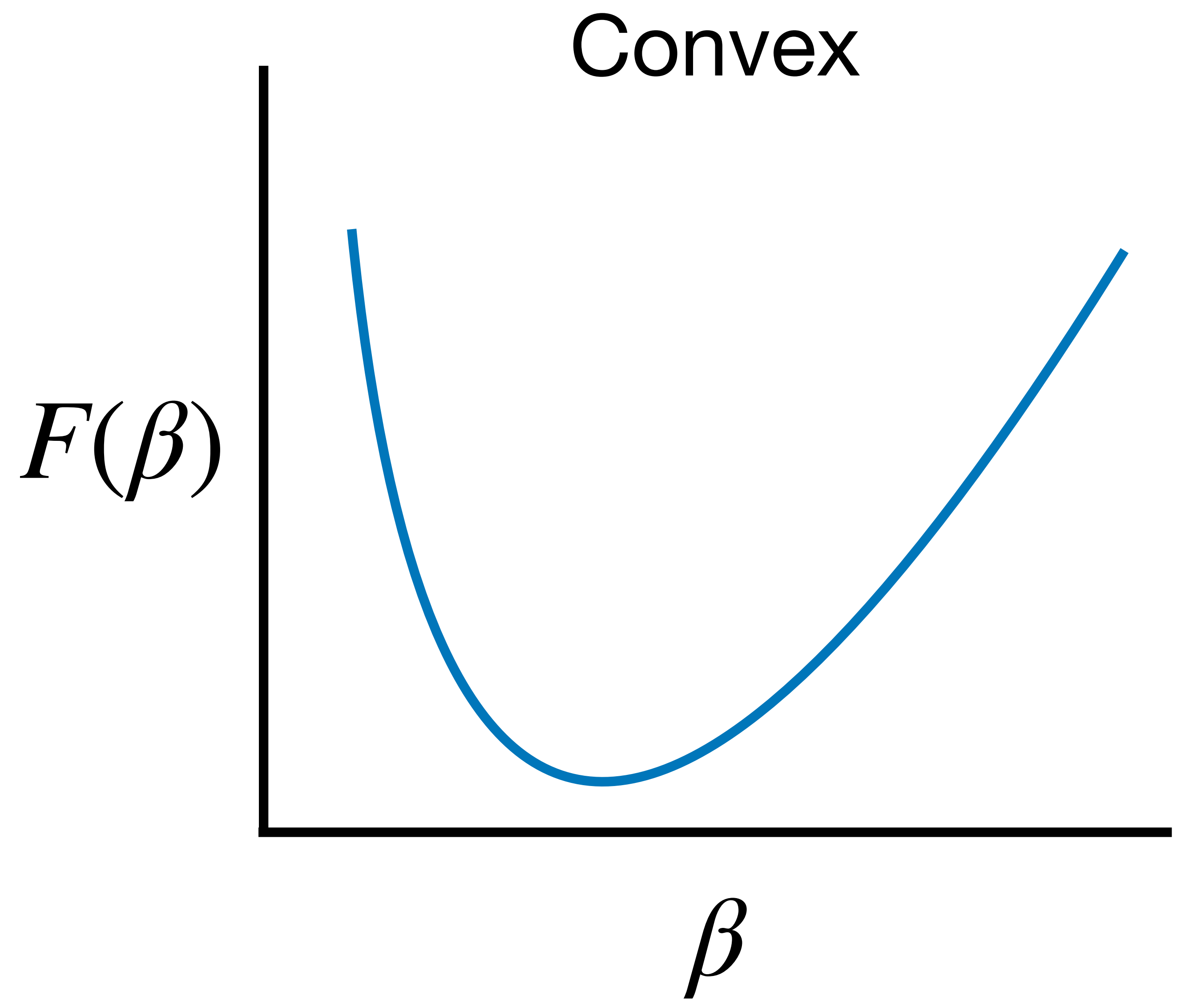

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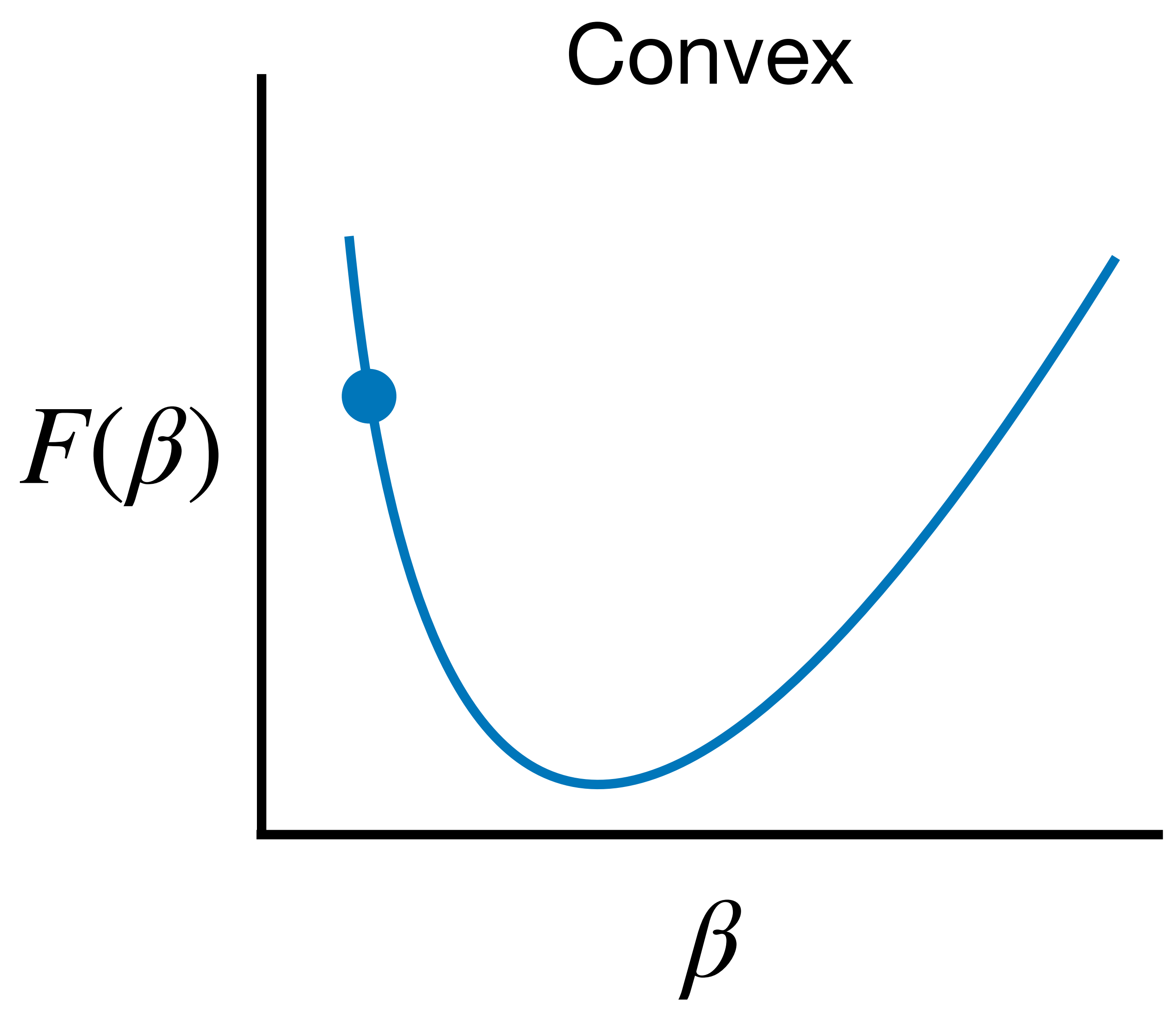

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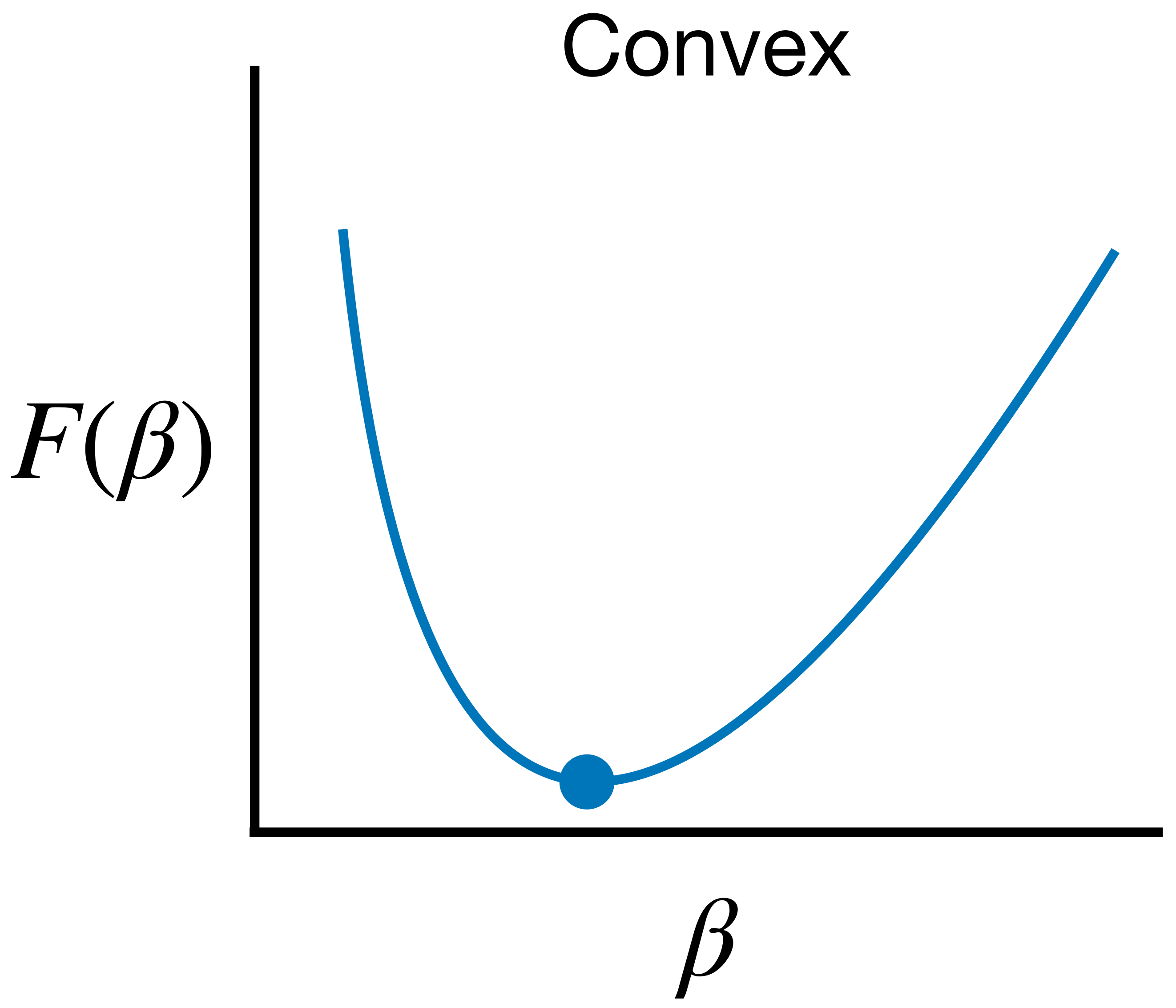

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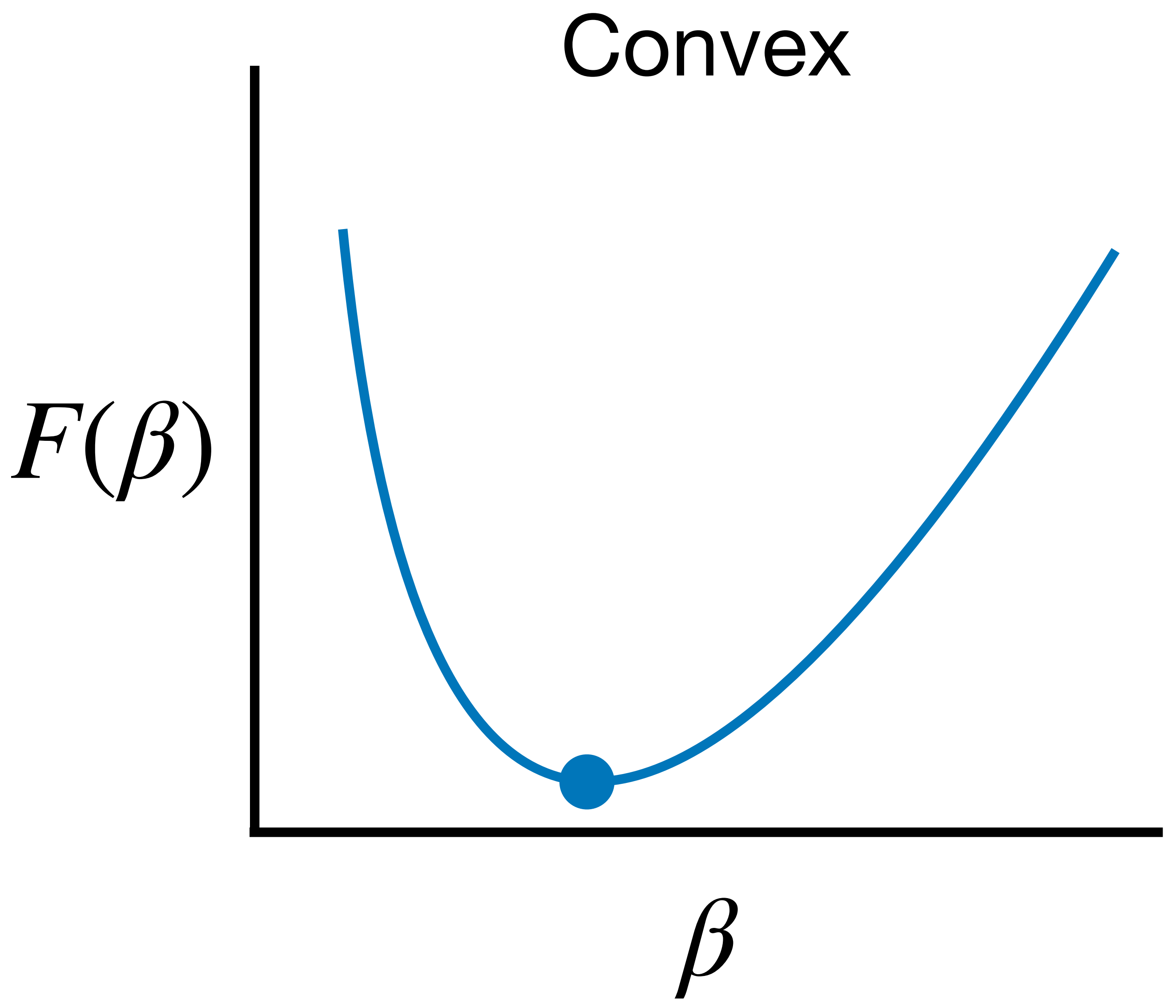

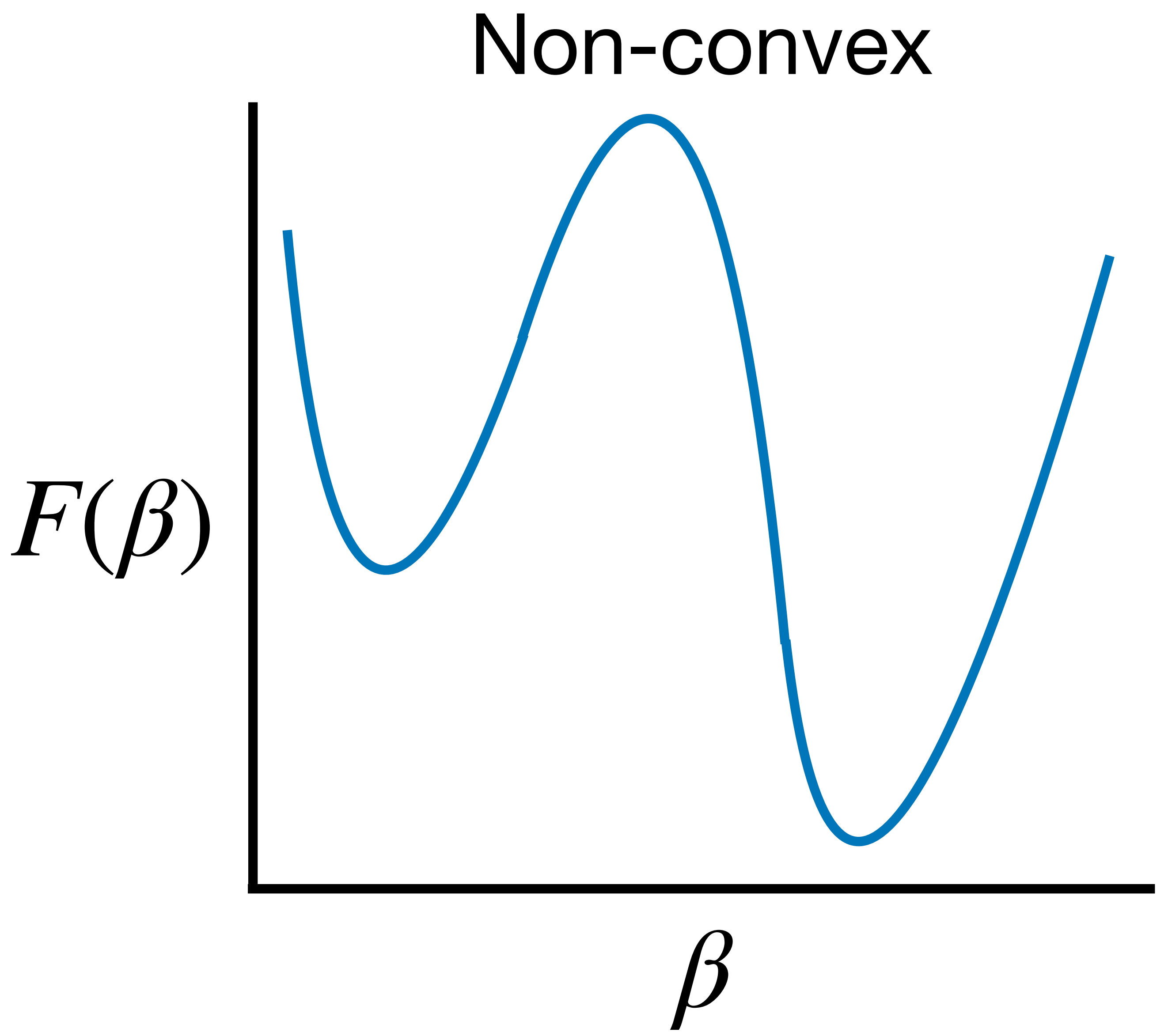

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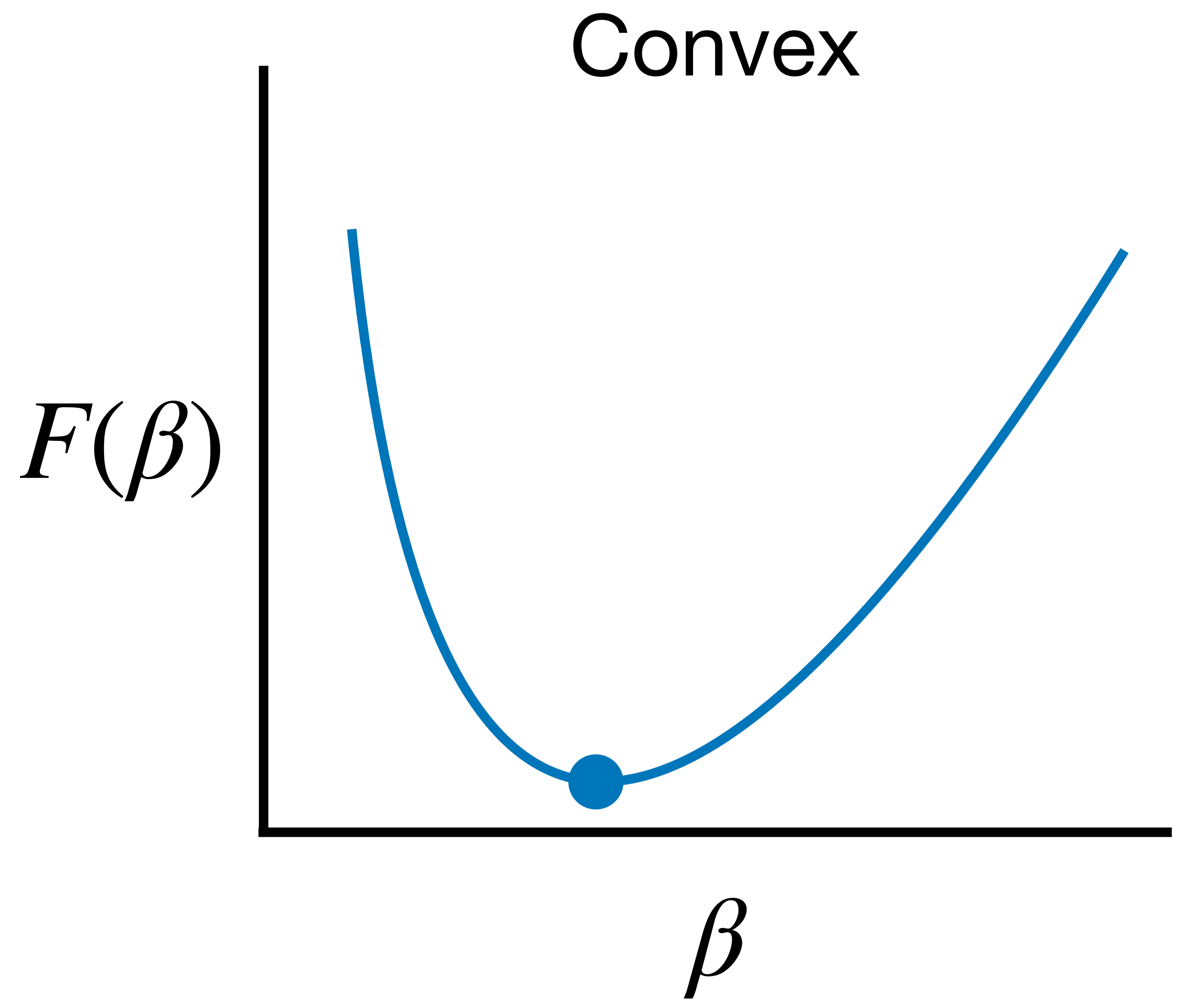

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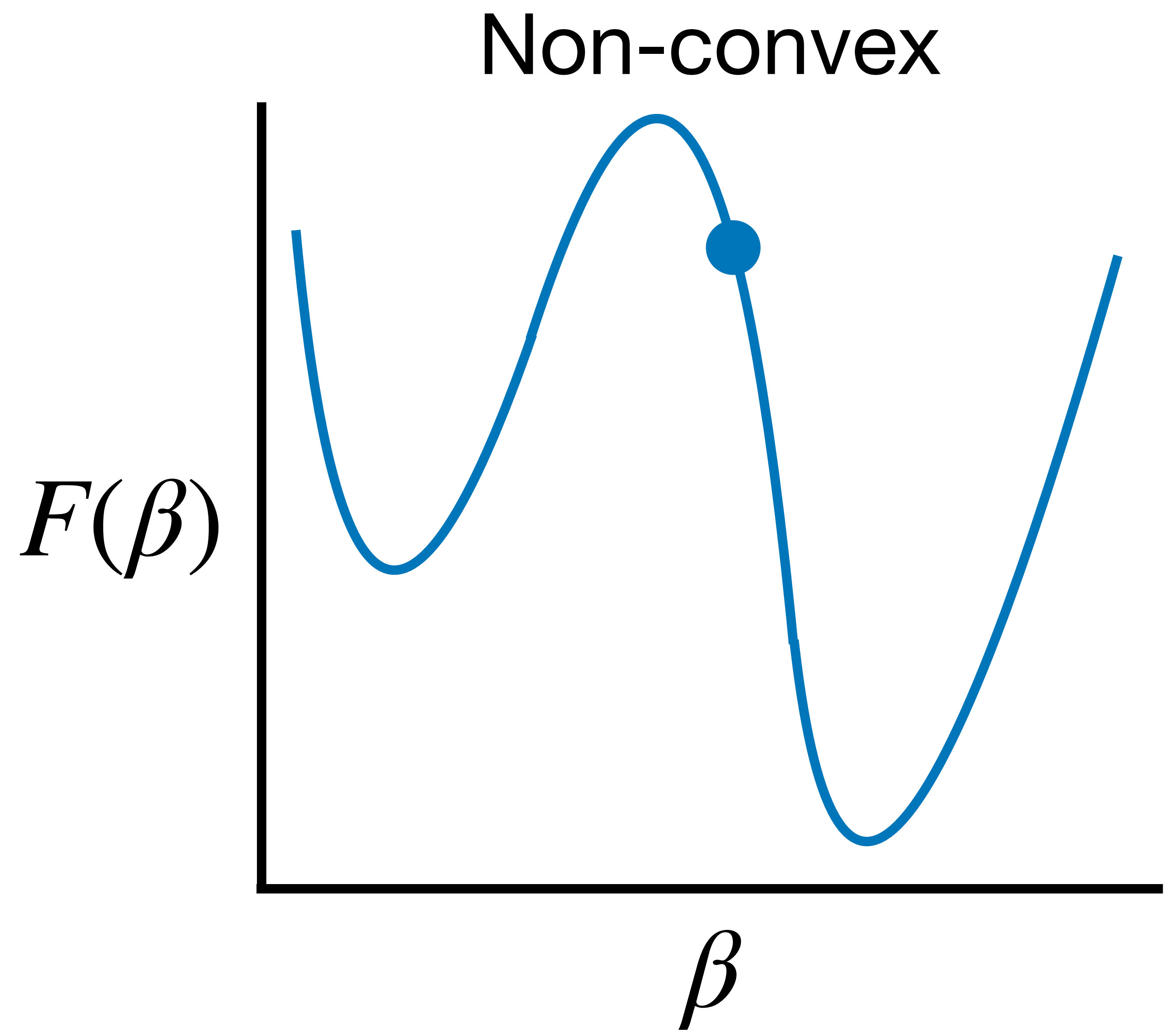

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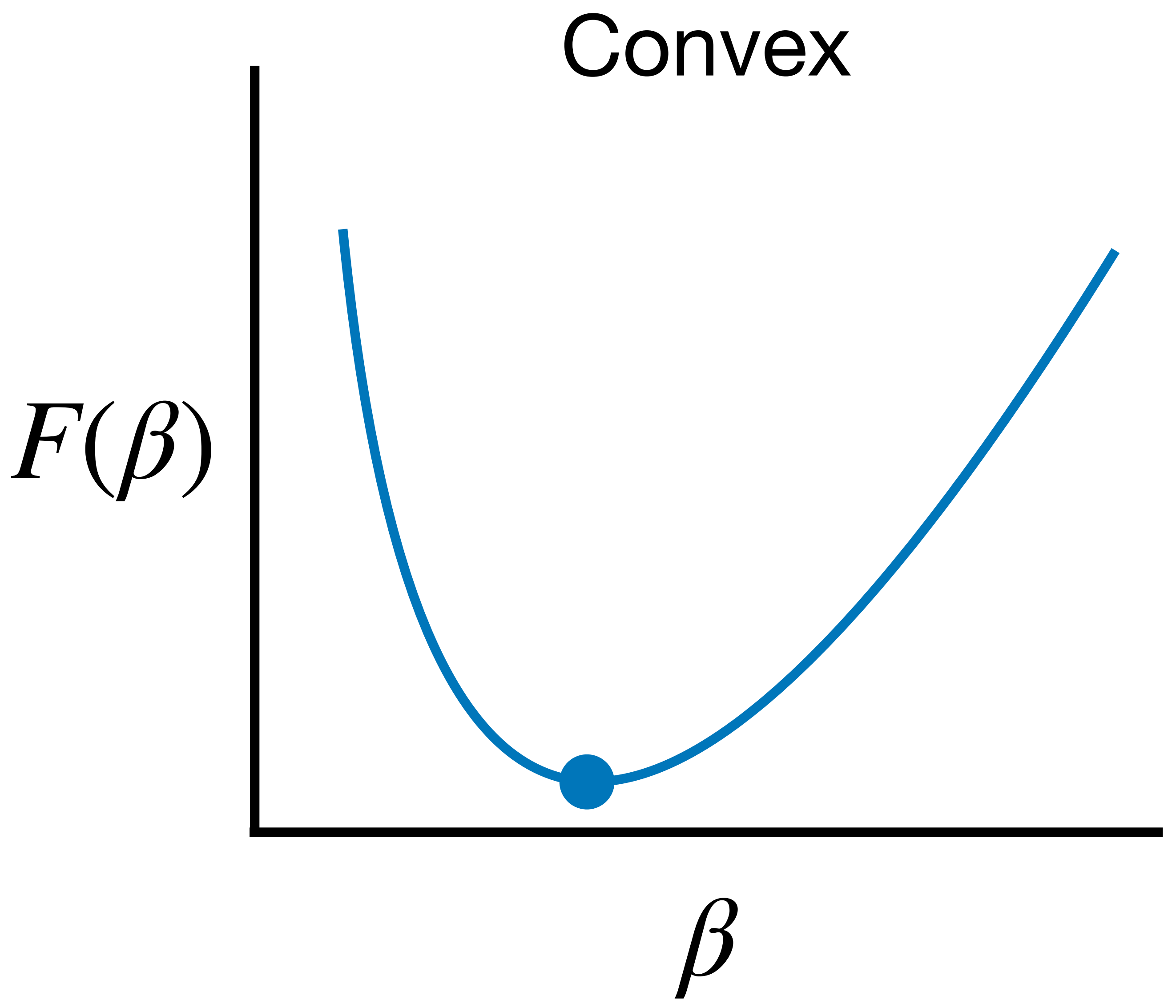

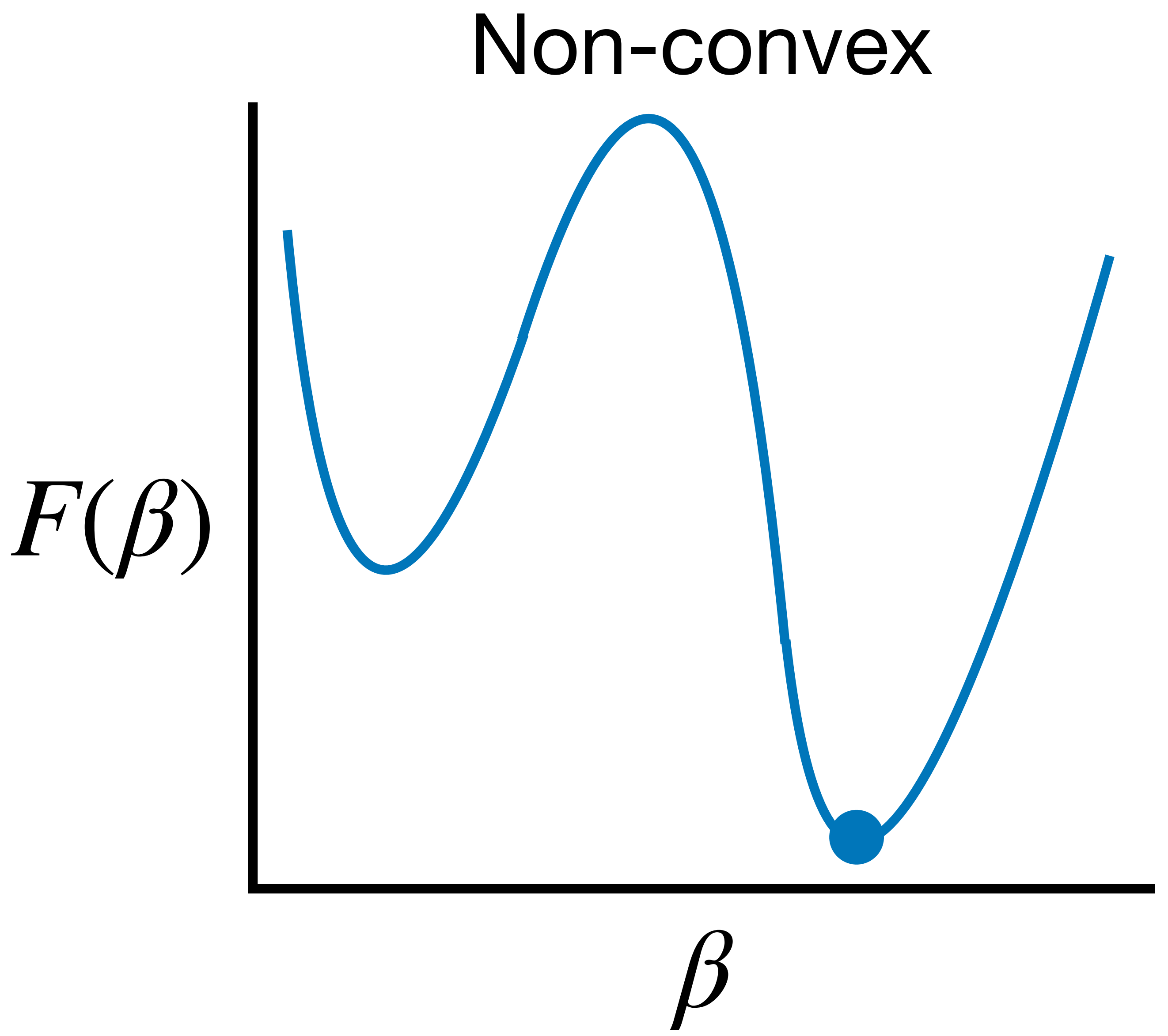

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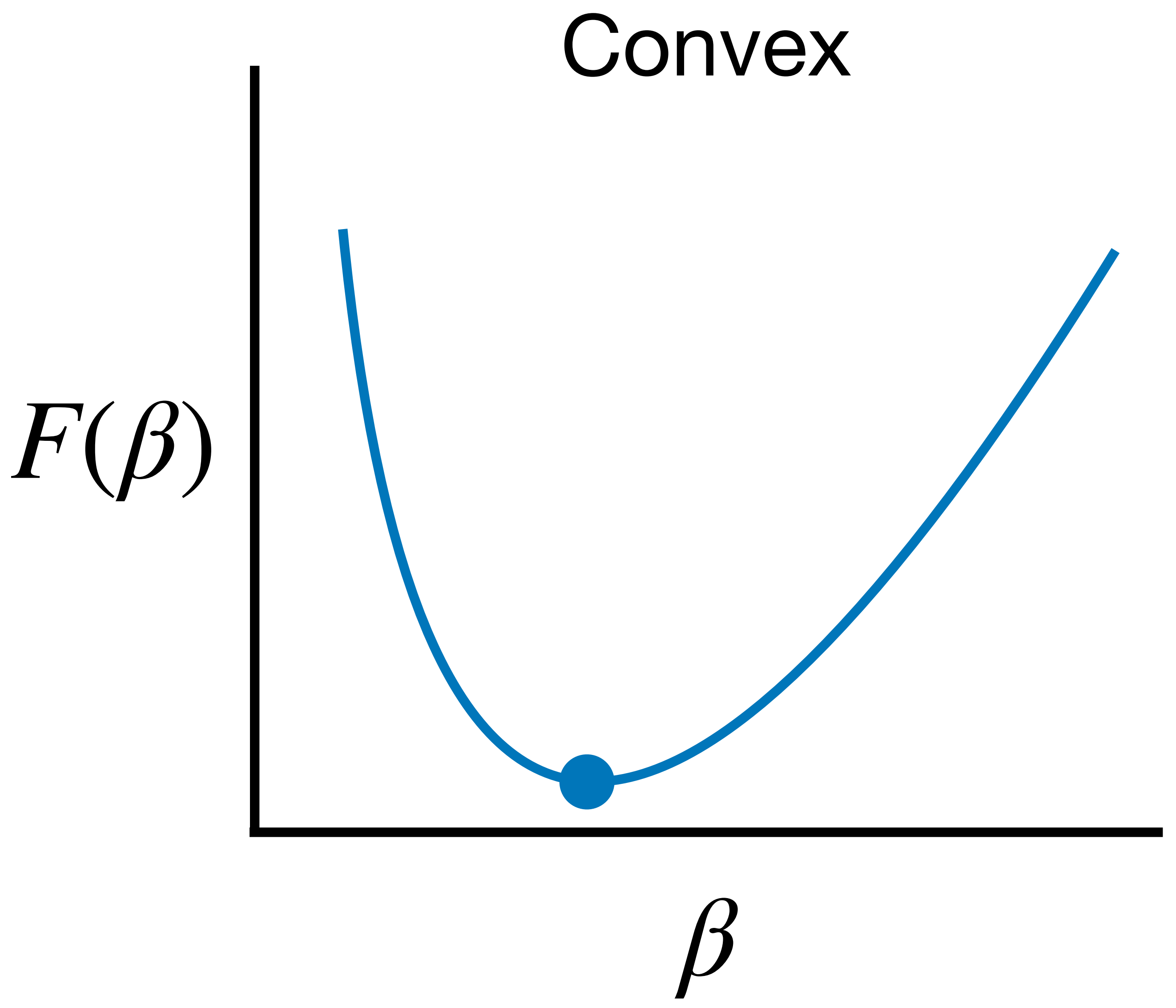

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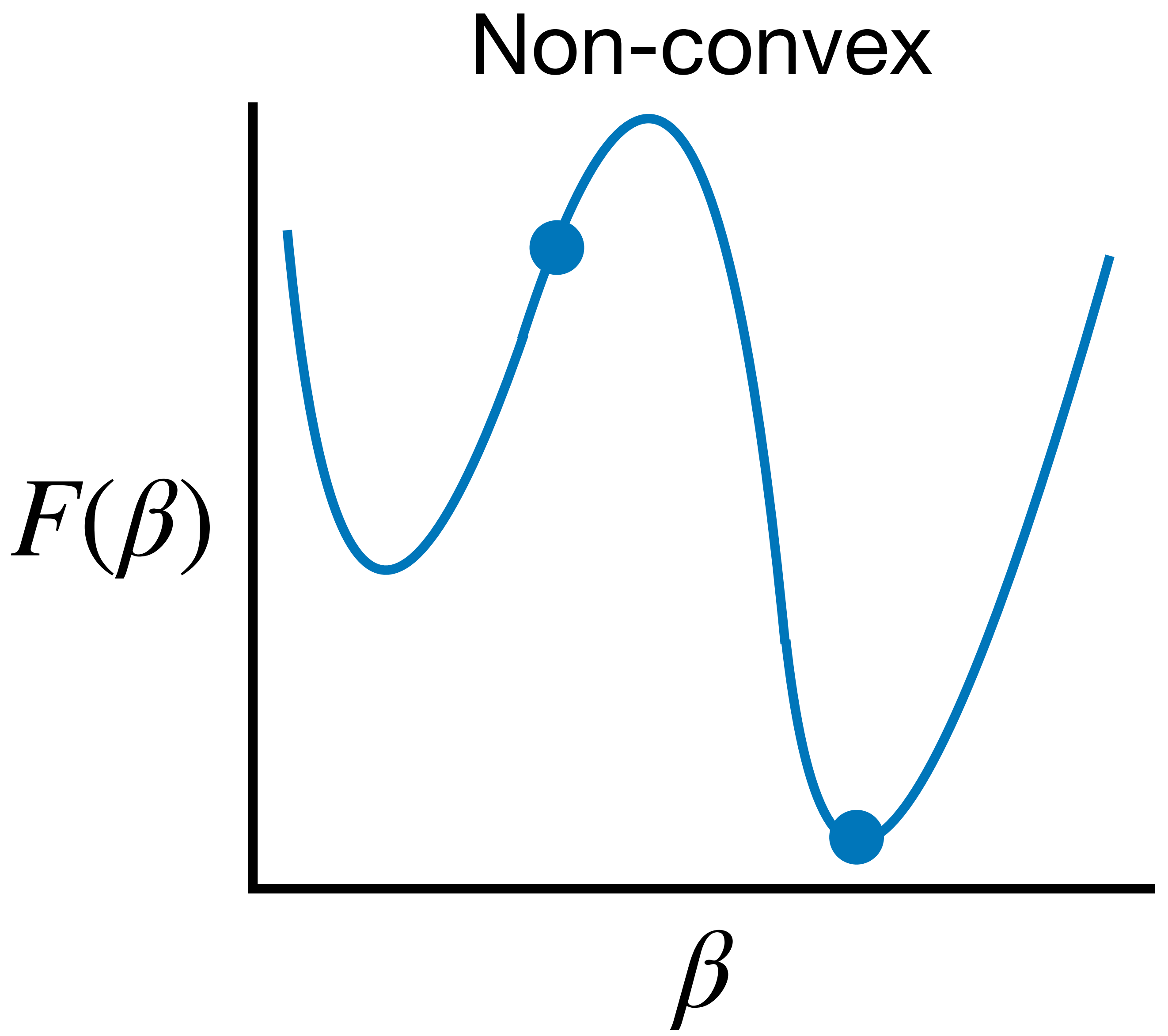

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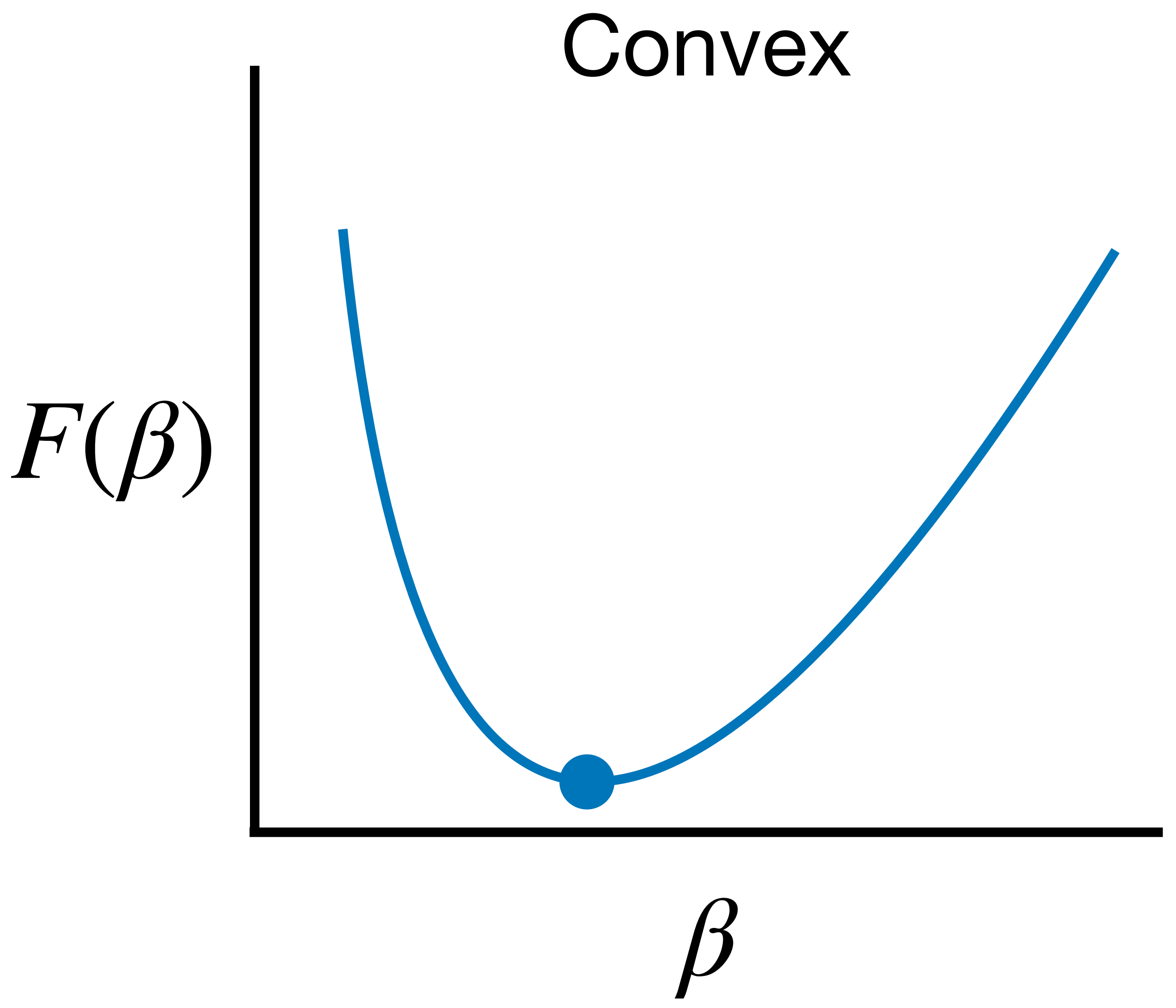

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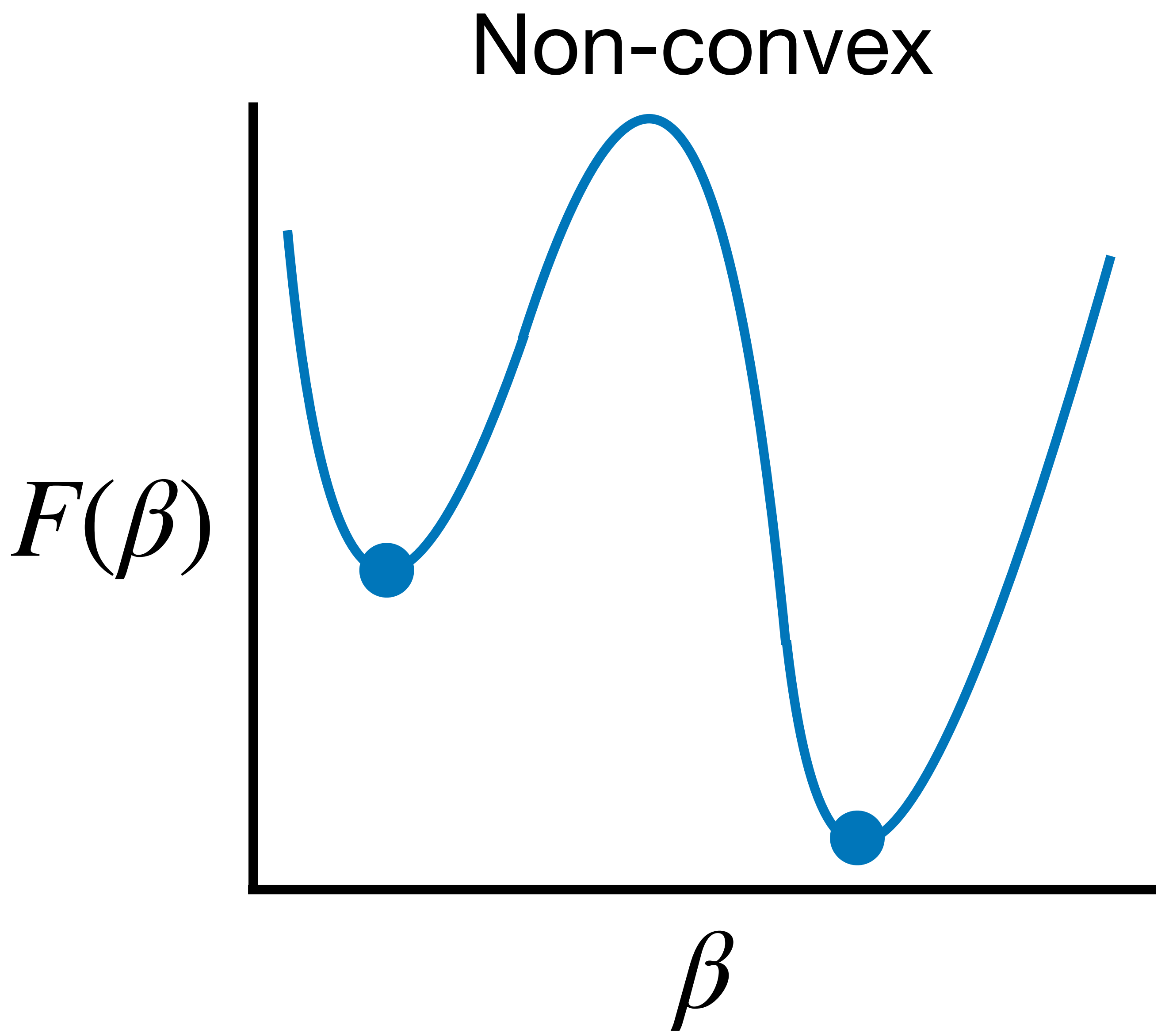

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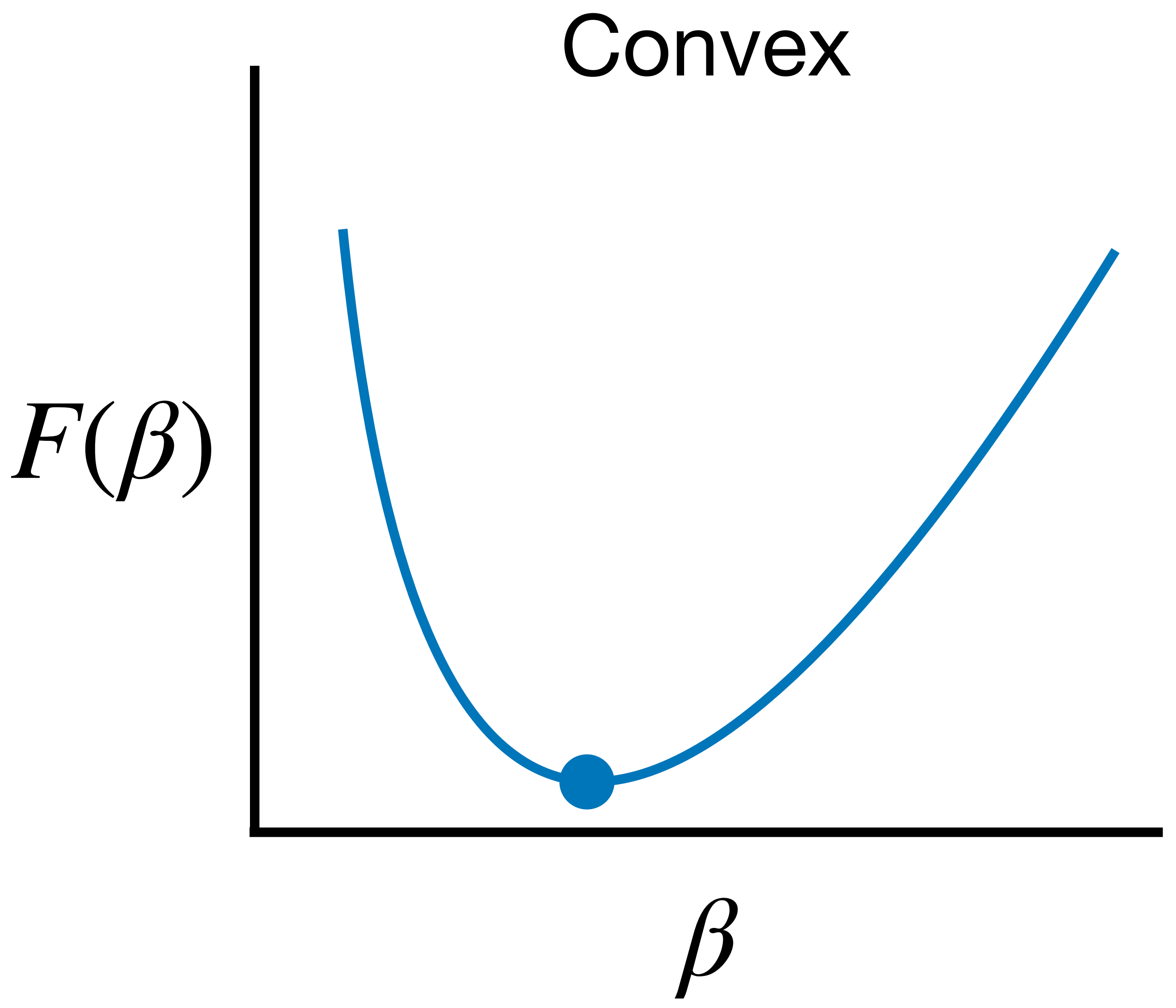

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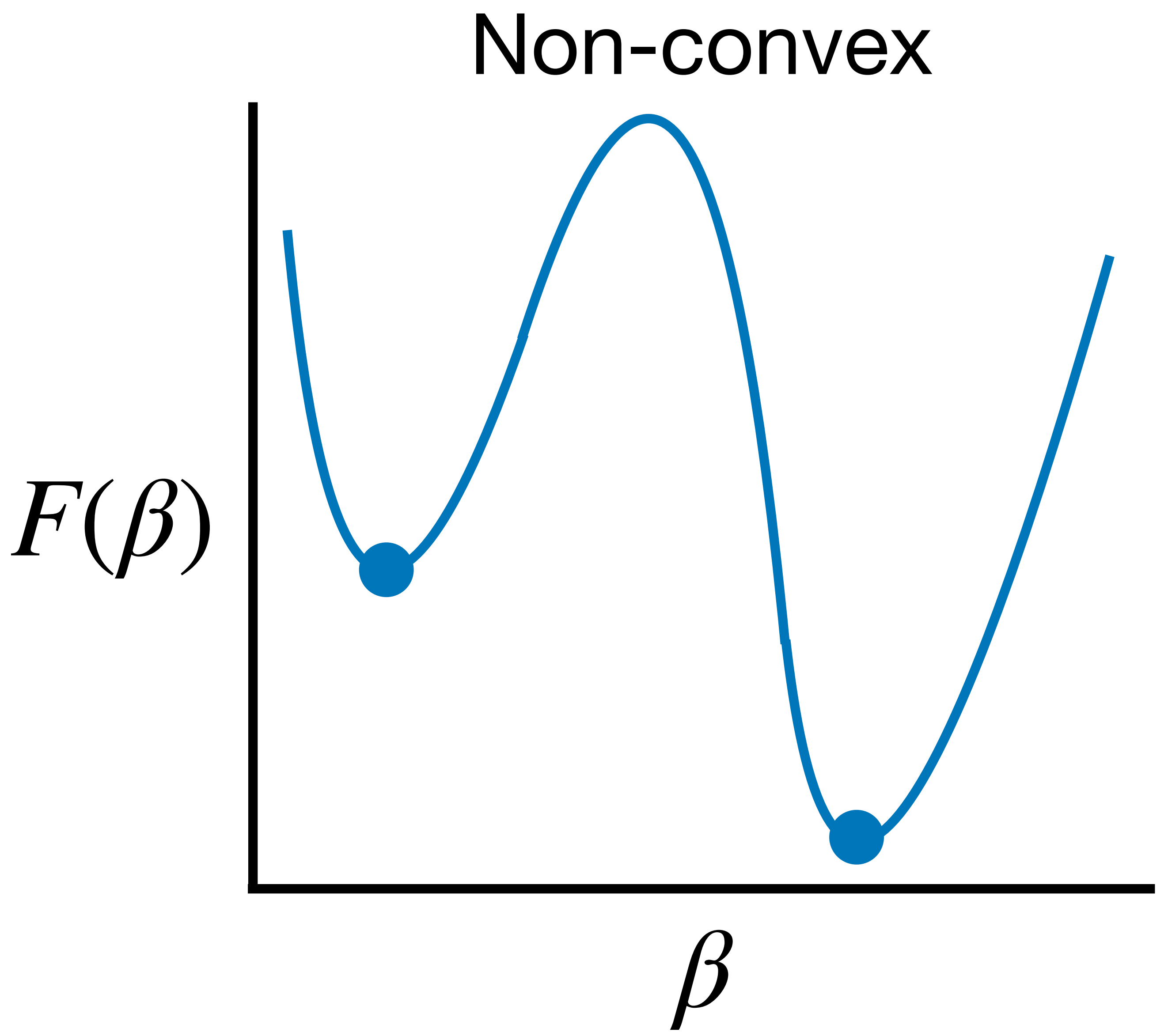

While it is computationally infeasible to find global minima for non-convex optimization,

- Local minima may still give reasonable models
- Other tricks, like multiple restarts, give better solutions

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- Solving optimization problems is a key part of training predictive models.
- Hardness of optimization depends on whether objective function is convex; linear and logistic regression are convex but trees and neural networks are not.
- Gradient descent is a common way to “go downhill” along an objective function, arriving at a local minimum (and for convex objectives, a global one).