## Boosting

STAT 4710

November 7, 2023

## Where we are

Unit 1: R for data mining
Unit 2: Prediction fundamentals
Unit 3: Regression-based methods
Unit 4: Tree-based methods
Unit 5: Deep learning

Lecture 1: Growing decision trees
Lecture 2: Tree pruning and bagging
Lecture 3: Random forests
Lecture 4: Boosting
Lecture 5: Unit review and quiz in class

## Looking back and looking ahead

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- Random forests: Grow deep decision trees in parallel
- Boosting: Grow shallow decision trees sequentially


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Final prediction rule: $\hat{f}=\hat{f}_{1}+\cdots+\hat{f}_{B}$.

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Large enough $B$ can lead to overfitting, unlike random forests.


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d=1: \quad \hat{f}\left(X_{1}, X_{2}, X_{3}\right)=\hat{g}_{1}\left(X_{1}\right)+\widehat{g}_{2}\left(X_{2}\right)+\hat{g}_{3}\left(X_{3}\right)
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Since each tree involves only one feature, the entire boosted model can be viewed as an additive model:


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\hat{f}(X)=\widehat{g}_{1}\left(X_{1}\right)+\hat{g}_{2}\left(X_{2}\right)+\cdots+\widehat{g}_{p}\left(X_{p}\right)
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for some coordinate functions $\widehat{g}_{j}$.
The coordinate functions can be easily plotted and interpreted.


## Derivation of coordinate functions

Get coordinate functions by grouping stumps splitting on the same variable:

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Subsampling increases variance of individual trees but de-correlates them; benefit of the latter tends to outweigh the former.

A subsampling fraction of $\pi=0.5$ tends to work well in most cases.

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The larger $d$ is, the worse the approximation.


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The key issue is that there is not an obvious notion of residual in classification.
Implementation of boosting for classification is beyond the scope of the class, but the same intuitions from this lecture carry over to boosting for classification.

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- Unlike random forests, boosting builds the trees sequentially rather than in parallel, using shallow trees rather than deep trees.
- Boosting works best when paired with shrinkage to further slow learning.
- Unlike random forests, the number of trees $B$ controls the complexity of the fit, and therefore must be tuned via cross-validation.


## Summary

- Like random forests, boosting aggregates the results of many decision trees to build a predictive model with state-of-the-art performance.
- Unlike random forests, boosting builds the trees sequentially rather than in parallel, using shallow trees rather than deep trees.
- Boosting works best when paired with shrinkage to further slow learning.
- Unlike random forests, the number of trees $B$ controls the complexity of the fit, and therefore must be tuned via cross-validation.
- Purity-based variable importance as well as partial dependence plots help interpret boosting models.

